Solutions to Math 477 "QUIZ" for Lecture 12

1. Buses arrive at a specified stop at 10-minute intervals, starting at 7 AM. It a passenger arrives at a time that is uniformly distributed between 7 AM and 8 AM, what is the probability that he would have to wait

(a) Less than 2 minutes?

(b) more than 9 minutes?

(c) Between 2 and 4 minutes if it is known that he had to wait less than 6 minutes.

Sol. to 1: Let X be the random variable 'time since the previous bus arrived'. The pdf is $f(x) = 1/10$ for $0 \leq x \leq 10$, and 0 otherwise.

a:

$$P\{8 \leq X \leq 10\} = \int_8^{10} \frac{1}{10}\, dx = \frac{2}{10} = \frac{1}{5} \quad .$$

b:

$$P\{0 \leq X \leq 1\} = \int_0^{1} \frac{1}{10}\, dx = \frac{1}{10} \quad .$$

c:

$$P(6 \leq X \leq 8 \,|\, 4 \leq X \leq 10) = \frac{P\{6 \leq X \leq 8\}}{P\{4 \leq X \leq 10\}} = \frac{2}{6} = \frac{1}{3} \quad .$$

2. The age-at-death, in years, of a certain population is normally distributed with mean 80 and variance 9.

Calculate the 30-th percentile of the age-at-death, in years.

Sol. to 2.: Let X be a normal r.v. with mean $\mu = 80$ and $\sigma = \sqrt{9} = 3$. Let a be the desired age, i.e. the age a such that %30 of the population die before a.

We need

$$P\{X \leq a\} = 0.3 \quad .$$

Converting to

$$Z = \frac{X - \mu}{\sigma} = \frac{X - 80}{3} \quad ,$$

that is a **standard** normal r.v (i.e. $N(0, 1)$) .

From $P\{X \leq a\} = 0.3$, we have (subtracting 80 from both sides)

$$P\{X - 80 \leq a - 80\} = 0.3 \quad .$$

Dividing by 3:

$$P\{\frac{X-80}{3} \leq \frac{a-80}{3}\} = 0.3 \quad .$$

This means

$$P\{Z \leq \frac{a-80}{3}\} = 0.3 \quad .$$

This means

$$\Phi(\frac{a-80}{3}) = 0.3 \quad .$$

Looking at the Z table, we get

$$\Phi(-0.525) = 0.3 \quad .$$

Hence

$$\frac{a-80}{3} = -0.525 \quad .$$

Solving for a:

$$a = -3 \cdot 0.525 + 80 = 78.425 \quad .$$

Ans. to 2: %30 of the population die before the age of 78.425 years old (so %70 live longer than that!).

Comment: The 80 is realistic (in many countries) but the $\sigma = 3$ is **not**, the actual s.d. is much higher.