Dr. Z.'s Intro to Probability Homework assignment 7

1 . If the probability mass function of the discrete random variable X is

$$P(X=0)=0.1 \quad , \quad P(X=1)=0.3 \quad , \quad P(X=2)=0.4 \quad , \quad P(X=3)=0.2 \quad ,$$

and $P(X=x)=0$ if $x \notin \{0,1,2,3\}$ find

(a) $E[2X^2-3X+1]$

(b) $E[sin(X)]$

2. If you are told that $E[X]=15$, find $E[11X-165]$.

3. The probability mass function for a certain discrete random variable X is as follows

$$P(X=1)=0.1 \quad , \quad P(X=3)=0.3 \quad , \quad P(X=6)=0.5 \quad , \quad P(X=11)=0.1 \quad ,$$

and $P(X=x)=0$ if $x \notin \{1,3,6,11\}$.

(a) Find the expectation, $E[X]$.

(b) Find the Variance $Var(X)$. Also find the standard deviation.

4. Let X be the winnings of a gambler and assume that

$$P(X=0)=\frac{1}{3} \quad ; \quad P(X=1)=\frac{13}{55} \quad ; \quad P(X=-1)=\frac{13}{55} \quad ;$$

$$P(X=2)=\frac{1}{11} \quad ; \quad P(X=-2)=\frac{1}{11} \quad ; \quad P(X=3)=\frac{1}{165} \quad ; \quad P(X=-3)=\frac{1}{165} \quad ;$$

Find the variance $Var(X)$. Also find the standard deviation.

5. The probability that you win i dollars $(1 \leq i \leq 5)$ is proportional to $\frac{1}{i}$.

(a) What is the probability of winning i dollars for $(1 \leq i \leq 5)$?

(b) Let X be the amount won, what is $Var(X)$? Also find the standard deviation.

6. An n-faced fair die, marked with $1, 2, \ldots n$ is rolled. Let X be the number of dots. Find $Var(X)$. Also find the standard deviation.

7. A life insurance policy is sold to a couple in case one or both of them die before the children reach the age of 21.

If one of them dies, it pays 100000 dollars, and if both of them die it pays 300000 dollars.

The probability of the father dying is 0.2, and the probability of the wife dying is 0.3, and are independent of each other.

Find the expectation and standard deviation of the amount paid by the insurance company?

8. An airport purchases an insurance policy to offset costs due to a large snowfall. For every full ten inches of snow in excess of 40 inches, during the winter season, the insurer pays the airport 300 up to a policy maximum of 700.

It is known that the probabilities of the amount of snowfall are as follows

- The probability that the snowfall is between 0 and 20 inches is: 0.06
- The probability that the snowfall is between 20 and 30 inches is: 0.18
- The probability that the snowfall is between 30 and 40 inches is: 0.26
- The probability that the snowfall is between 40 and 50 inches is: 0.22
- The probability that the snowfall is between 50 and 60 inches is: 0.14
- The probability that the snowfall is between 60 and 70 inches is: 0.06
- The probability that the snowfall is between 70 and 80 inches is: 0.04
- The probability that the snowfall is between 80 and 90 inches is: 0.04
- The probability that the snowfall is more than 90 inches is: 0.

Calculate the standard deviation of the amount paid under the policy.