

Dr. Z.'s Intro to Probability Homework assignment 5

Version of Sept. 21, 2017 (correcting a typo in #4, pointed out by Zhiyun Xu)

1. A loaded coin, whose probability of Heads is 0.7 is tossed 10 times. Assuming that the tosses are independent, what is the probability that

(i) it landed Heads all the time ?

(ii) it landed Heads 9 times and Tails one time?

2. If you roll a loaded die, whose probabilities are $P(1) = 0.1, P(2) = 0.2, P(3) = 0.3, P(4) = 0.1, P(5) = 0.25, P(6) = 0.05$, ten times, what is the probability that five of them are divisible by 3, two of them give you remainder 1 when dividing by 3 (and hence the remaining three give you remainder 2 when dividing by 3)?

3. Assume that the probability of being lazy, of being smart, and of being strong, are all independent of each other. If the probability of being lazy is 0.4, of being smart is 0.2 and the probability of being all of them is 0.01, what is the probability of being strong?

4. A certain language has 4 letters, $\{A, B, C, D\}$. The frequency of A is 0.4, the frequency of B is 0.3, the frequency of C is 0.2, the frequency of D is 0.1. If you make a random word of 10 letters (and draw each letter independently of the other ones), what is the probability that it has 3 A 's, 2 B 's, 2 C 's, and 3 D 's?

5. It is known that the probability of being lazy is twice the probability of being smart, and the probability of being smart is three times the probability of being strong. All three traits are independent of each other. If the probability of being lazy and smart and strong is $\frac{9}{500}$, what are the probabilities of being lazy?, of being smart?, of being strong?

6. A company issues auto insurance policies. There are 900 insured individuals. Fifty-four percent of them are male. If a female is randomly selected from the 900, the probability that she is over 25 years old is 0.43.

There are 395 total insured individuals over 25 years old.

A person less-than-or-equal to 25 years old is randomly selected.

Calculate the probability that the person selected is male.

7. A company takes out an insurance policy to cover accidents that occur in its manufacturing plant. The probability that one or more accidents will occur during any given month is $\frac{3}{5}$. The number of accidents that occur in any given month is independent of the number of accidents that occur in all other months.

Calculate the probability that there will be at least four months in which no accidents occur before

the fourth month in which at least one accident occurs.