

Dr. Z.'s Intro to Probability Homework assignment 23

Version of Dec. 2, 2017 (please discard all previous versions)

1. State the Central Limit Theorem.

2. A carpenter wants to measure the length of a table. Each measurement is independent of the others, and have a common mean d and common standard-deviation of 0.01 cm. He estimates the length of the table by taking the average of all the measurements. How many measurements does he have to make in order to make %95 sure that his estimate is accurate to within ± 0.001 cm? if he trusts the Central Limit Theorem? How many would he have to make if he uses the more conservative Chebyshev inequality?

3. If 30 fair dice are rolled, find the approximate probability that the average number of dots is between 2 and 4.

4. Let X_1, \dots, X_{25} be independent random variables, each of which is uniformly distributed on $[0, 2]$, and let $X = \sum_{i=1}^{25} X_i$.

(i): Find the mean and variance of X .

(ii): Use the Central Limit Theorem to estimate the probability that $|X - 24| \leq 3$.

5. Let $X_i, i = 1, \dots, 20$, be independent, identically distributed, continuous random variables each with probability density function is

$$f(x) = \begin{cases} 3x^2, & \text{if } 0 < x < 1; \\ 0, & \text{otherwise.} \end{cases}$$

Calculate an approximation to

$$P \left\{ \sum_{i=1}^{20} X_i < 16 \right\} .$$

6. State the Strong Law of Large Numbers.