Dr. Z.'s Intro to Probability Homework assignment 22

Version of Dec. 17, 2017 (Thanks to Jack Chiu)

1. State and prove Markov's inequality.

2. Let X be the random variable whose probability density function is

$$f(x) = \begin{cases} \frac{24}{7\,x^4}\,, & if \quad 1 \le x \le 2\,; \\ 0\,, & otherwise. \end{cases} \quad .$$

Verify Markov's inequality for $a = 1.5$ by finding $P\{X \ge 1.5\}$ and $E[X]/1.5$.

3. State and prove Chebyshev's inequality.

4. If X is the uniform distribution on $(0, 100)$,

(a) What is the probability that X is between 10 and 90?

(b) What does Chebyshev's inequality tells you about this probability?

5. Let X be the random variable whose probability density function is

$$f(x) = \begin{cases} \frac{24}{7\,x^4}\,, & if \quad 1 \le x \le 2\,; \\ 0\,, & otherwise. \end{cases} \quad .$$

First find the mean μ and the standard deviation σ. Then verify Chebyshev's inequality for $k = 2\sigma$.

6. Suppose that it is known that the amount of gold dug in a Gold mine during one day is a random variable with mean 10 kg.

(a) What can be said about the probability that the day's production will exceed 11 kg?

(b) If the standard deviation of the amount of gold dug in the Gold mine equals 1 kg, what can be said about the day's production will be between 8 and 12 kg?

7. State and prove the weak law of large numbers.