Dr. Z.'s Intro to Probability Homework assignment 21

1. The value of a piece of factory equipment after three years of use is $100(0.5)^X$, where X is a random variable having moment generating function

$$M_X(t) = \frac{1}{1-2t} \quad , \quad for \quad t < \frac{1}{2} \quad .$$

Calculate the expected value of this piece of equipment after three years of use.

2. Find the first two moments, and the variance, of a random variable X whose moment generating function is given by

$$M_X(t) = (1-2t)^{-4} \quad .$$

3. An actuary determines that the claim size for a certain class of accidents is a random variable X, with moment generating function

$$M_X(t) = \frac{1}{(1-2500t)^4} \quad .$$

Determine the standard deviation of the claim size for this class of accidents.

4. Let X and Y be identically distributed independent random variables such that the moment generating function of $X+Y$ is

$$M(t) = 0.09e^{-2t} + 0.24e^{-t} + 0.34 + 0.24e^{t} + 0.09e^{2t} \quad , \quad for \quad -\infty < t < \infty \quad .$$

Calculate $P\{X \leq 0\}$.

5. A company insures homes in three cities, J, K and L. Since sufficient distance separates the cities, it is reasonable to assume that the losses occurring in these cities are independent.

The moment generating functions for the loss distributions of the cities are

$$M_J(t) = (1-2t)^{-3} \quad ,$$
$$M_K(t) = (1-2t)^{-2.5} \quad ,$$
$$M_L(t) = (1-2t)^{-4.5} \quad .$$

Let X represent the combined losses from the three cities. Calculate $E(X^3)$

6. X and Y are independent random variables with common moment generating function $M(t) = e^{t^2/2}$.

Let $W = X+Y$ and $Z = Y - X$.

Determine the joint moment generating function, $M(t_1, t_2)$ of W and Z.