Dr. Z.'s Intro to Probability Homework assignment 20

1. In a certain community the maximum number of boys is 3 and the maximum number of girls is 3.

It is found that the probability density function

$$p(i,j) = Pr(NumberOfBoys = i, NumberOfGirls = j) = \frac{c}{1+i+j} \quad , \quad 0 \le i \le 3 \quad , \quad 0 \le j \le 3 \quad .$$

(a) Find the expected number of girls in families with i boys for $i = 0, 1, 2, 3$.

(b) Find the expected number of boys in families with j girls for $j = 0, 1, 2, 3$.

(c) Find the expected number of girls if it is known that there are at least as many girls as boys .

(d) Find the expected number of boys if it is known that there are at least as many girls as boys .

Leave your answers as Maple commands. If you have Maple, please compute them.

2. Suppose that the joint density of X and Y is given by

$$f(x,y) = \begin{cases} x+y & , \quad if \quad 0 < x < 1, 0 < y < 1; \\ 0 & , \quad otherwise. \end{cases} \quad .$$

Find

(a) $E[X \mid Y = y]$

(b) $E[Y \mid X = x]$

3. Suppose that the joint density of X and Y is given by

$$f(x,y) = \begin{cases} \frac{12(x^2+y)}{5} & , \quad if \quad 0 < y < x < 1; \\ 0 & , \quad otherwise. \end{cases} \quad .$$

Find (a) $E[X \mid Y = y]$; (b) $E[Y \mid X = x]$.

Leave your answers as Maple commands. If you have Maple, please compute them.

4. Suppose that the joint density of X and Y is given by

$$f(x,y) = \begin{cases} \frac{12(x^2+y)}{5} & , \quad if \quad 0 < y < x < 1; \\ 0 & , \quad otherwise. \end{cases}$$

Find (a) $E[X \mid X + Y < 1]$; (b) $E[Y \mid X + Y < 1]$.

Leave your answers as Maple commands. If you have Maple, please compute them.

5. Two life insurance policies, each with a death benefit of $10,000$ and a one-time premium of 500, are sold to a couple, one for each person. The policies will expire at the end of the tenth year. The probability that only the wife will survive at least ten years is 0.025, the probability that only the husband will survive at least ten years is 0.01, and the probability that both of them will survive at least ten years is 0.96,

What is the expected excess of premiums over claims, given that the husband survives at least ten years?

6. A miner is trapped in a mine containing 4 doors.

• The first door leads to a tunnel that will take him to safety after 2 hours of travel.

• The second door leads to a tunnel that will take him back to the mine in 3 hours of travel.

• The third door leads to a tunnel that will take him back to the mine in 4 hours of travel.

• The fourth door leads to a tunnel that will take him back to safety in 5 hours of travel.

If the probabilities of him choosing the first, second, third, and fourth doors are $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{8}$ respectively, what is the expected length of time until he reaches safety?