Dr. Z.'s Intro to Probability Homework assignment 19

1. Suppose that

- Alice is 160cm -tall, has an IQ 120 and weighs 50kgs ;

- Barbara is 170cm -tall, has an IQ 130 and weighs 60kgs ;

- Courtney is 155cm -tall, has an IQ 115 and weighs 55kgs.

Assuming that each girl is equally likely to be picked, find the correlation between height and weight, height and IQ, and weight and IQ.

2. In a certain family of three girls, the scores (out of 100) for English and Math are as follows

Alice: English 100 ; Math: 60

Barbara: English 90 ; Math: 70

Courtney: English 80 ; Math: 80

What is The correlation between the English scores and the Math scores?

3. In the sample space $\{1,2,3\}$, with $Pr(i) = \frac{1}{3}$, define the random variables $X(i) = i, Y(i) = 4-i$.

(a) Find $E[X]$ and $E[Y]$.

(b) Find $Var(X)$ and $Var(Y)$.

(c) Find the covariance $Cov(X,Y)$ and the correlation $\rho(X,Y)$.

4. If $Var(X) = 100$, $Var(Y) = 200$, and $Var(X+Y) = 400$, find

(i) $Var(2X+3Y)$;

(ii) $Var(X-Y)$.

5. Suppose that $Var(X) = 1, Var(Y) = 2, Var(Z) = 3$, $Var(X+Y) = 4, Var(X+Z) = 5, Var(Y+Z) = 6$. Find

(i) $Var(X+Y+Z)$;

(ii)$Var(2X-Y+Z)$.

6. An insurance policy pays a total medical benefit consisting if two parts for each claim. Let X represent the part of the benefit that is paid to the surgeon, and let Y represent the part that is paid to the hospital. The variances of X is 5000, the variance of Y is $10,000$, and the variance of the total, $X+Y$, is $17,000$.

Due to increasing medical costs the company that issues the policy decides to increase X by a finite amount of 100 per claim, and to increase Y by 10% per claim.

Calculate the variance of the total benefit after these revisions are made.

7. Let X and Y be the number of hours that a randomly selected person watches movies and sports events, respectively, during a three-month period. The following information is known about X and Y.

$$E[X] = 50 \quad , \quad E[Y] = 20 \quad , \quad Var(X) = 50 \quad , \quad Var(Y) = 30 \quad , \quad Cov(X, Y) = 10 \quad .$$

One hundred people are randomly selected and observed for these three months. Let T be the total number of hours that these one hundred people watch movies or sports events this three month period.

Approximate the value of $P(T < 7100)$.