Dr. Z.'s Intro to Probability Homework assignment 18

1. You go to a strange casino where you have a chance of 0.01 to win 100 dollar, and 0.99 chance to lose a dollar. You do it for n days, and each time is independent of the other times. If X is the random variable denoting your gain, what is the probability generating function? What is $E[X]$? What is $Var(X)$?

2. Come up with a pair of six-faced dice with different number of dots, such that the distribution of the sum of the dots is the same as the distribution of throwing two standard dice (labeled $1, 2, 3, 4, 5, 6$), assuming that each face is equally likely to show up. **Prove that it works**!

(Hint, added Nov. 13, 2017: google "Sicherman dice")

3. I am walking up a stair-case, with probability $\frac{3}{4}$ I go to the next stair, with probability $\frac{1}{8}$ I walk two stairs at once, and with probability $\frac{1}{8}$ I walk three stairs at once. Each step is independent of the other ones. Let X be the random variable, "number of stairs climbed in n steps". What is the probability generating function of X? What is $E[X]$, What is $Var(X)$?

4. State the Gambler's Ruin problem for a fair casino, where you win a dollar with probability $\frac{1}{2}$ and you lose a dollar with probability $\frac{1}{2}$. Derive (with proof) the expressions for the probability of exiting a winner, and for the expected duration of the game, where the initial capital is x dollars, and you play until you are either broke or make N dollars.

5. State the Gambler's Ruin problem for an unfair casino, where you win a dollar with probability p and you lose a dollar with probability $q = 1 - p$. Derive (with proof) the expression for the probability of exiting a winner, where the initial capital is x dollars, and you play until you are either broke or make N dollars.

6. If you enter a casino with 700 dollars, and wish to make 1000 dollars, and the probability, at each round, of winning a dollar is 0.48 and losing a dollar is 0.52, what is the probability of exiting a loser?