Dr. Z.'s Intro to Probability Homework assignment 16

Version of Nov. 13, 2017: Please discard previous versions (if they exist).

1. In a certain community, the probability that a family has i boys and j girls is given by

$$p(i,j) = \begin{cases} \frac{c}{(i+j+1)^2} \quad , & if \quad 0 \leq i \leq 2 \quad and \quad 0 \leq j \leq 3; \\ 0 \quad , & otherwise. \end{cases} \quad ,$$

for some positive constant c (that would make it a discrete probability mass function). Calculate the conditional probability mass function for the number of boys in families that have exactly 2 girls.

2. In a certain (mostly) wealthy town, the probability that a household has i cars and j bed-rooms is

$$p(i,j) = \begin{cases} \frac{2\,i+3\,j}{744} & if \quad 0 \leq i \leq 5 \quad and \quad 0 \leq j \leq 7; \\ 0 \quad , & otherwise. \end{cases} \quad ,$$

If it is known that a household has 5 bed-rooms, what is the probability that it has at least two cars?

3. The joint density function of X and Y is given by

$$f(x,y) = \begin{cases} x+y \quad , & if \quad 0 < x < 1 \, , 0 < y < 1 \, ; \\ 0 \quad , & otherwise, \end{cases}$$

(i) Compute the conditional density of X given that $Y = y$. (ii) If you know that $Y = 0.6$ what is the probability that $0.4 \leq X \leq 0.7$.

4. The joint density function of X and Y is given by

$$f(x,y) = \begin{cases} 3x + 3y \quad , & if \quad 0 < x \; , \; 0 < y, \; and \; x+y < 1 \, ; \\ 0 \quad , & otherwise. \end{cases} \quad .$$

(i) Compute the conditional density of X given that $Y = y$ and the conditional density of Y given that $X = x$.

(ii) If it is known that $Y = \frac{1}{4}$, what is the probability that $X \leq \frac{1}{2}$?

5. A company offers a basic life insurance policy to its employees, as well as a supplemental life insurance policy. To purchase the supplemental policy, an employee must first purchase the basic policy.

Let X denote the proportion of employees who purchase the basic policy, and let Y the proportion of the employees who purchase the supplemental policy. Let X and Y have joint density function $f(x,y) = 2(x+y)$ on the region where the density is positive.

Given that 10% of the employees buy the basic policy, determine the probability that fewer than 5% buy the supplemental policy.

6. In a certain community of married couples, the maximal income of the wife is 300K and the maximal income of the husband is 100K. Every husband makes **at most** a third of his wife's income. Let X denote the the wife's income and let Y denote the husband's income. Let X and Y have joint density function $f(x, y) = 2(x + y)/7$ on the region where the density is positive. The unit of money is 100K.

If it is known that the wife makes 240000 dollars, what is the probability that the husband makes more than 60000 dollars?