Dr. Z.'s Intro to Probability Homework assignment 15

1. Two friends decide to meet at a certain restaurant. If each of them independently arrives at a time uniformly distributed between 1pm and 1 : 20pm. Find the probability that the first to arrive has to wait longer than 5 minutes.

2. Two friends decide to meet at a certain restaurant at 1:30pm. One of them, Mr. A, has a propensity to arrive earlier than the agreed time. The density function of his arrival time is

$$a(x) = \begin{cases} \frac{15-x}{450}, & if \quad -15 < x < 15\,; \\ 0, & otherwise \end{cases} .$$

(x is measured in minutes)

The other friend, Mr. B, has a propensity to arrive later than the agreed time. The density function of his arrival time is

$$b(x) = \begin{cases} \frac{x+15}{450}, & if \quad -15 < x < 15\,; \\ 0, & otherwise \end{cases} .$$

Find the probability that Mr. A has to wait longer than 10 minutes for Mr. B.

3. Let X and Y be independent uniform distributions $U(0,1)$. Prove that the density function of $X+Y$ is

$$f_{X+Y}(a) = \begin{cases} a \quad if \quad 0 < a < 1 \quad ; \\ 2-a \quad if \quad 1 < a < 2 \quad ; \\ 0 \quad otherwise. \end{cases} .$$

4. Let X and Y be independent continuous random variables with densities

$$f_X(x) = \begin{cases} 2e^{-2x}, & if \quad x \geq 0\,; \\ 0 & otherwise. \end{cases} \quad , \quad g_Y(y) = \begin{cases} 5e^{-5y}, & if \quad y \geq 0\,; \\ 0 & otherwise. \end{cases} .$$

Find the density function $f_Z(z)$ for $Z = X+Y$.

5. The number of spam emails that I get in my Gmail account has a Poisson distribution with mean 1. The number of spam emails that I get in my Rutgers account has a Poisson distribution with mean 3. They are independent of each other. What is the probability that the total number of spam emails is 5?

6. Let X_1, X_2, X_3, X_4 be **independent** Normal distributions with means $1, 2, 3, 4$ respectively and standard-deviations $2, 3, 4, 5$. What is the mean of their average $(X_1+X_2+X_3+X_4)/4$. What is its standard deviation?

7. Let X_i $1 \leq i \leq n$ be **independent** Poisson distributions with parameters μ_i. What is the probability that $X_1 + \ldots + X_n$ is exactly 3?