Dr. Z.'s Intro to Probability Homework assignment 14

1. In a certain community the maximum number of boys and the maximum number of girls are both 3.

It is found that the probability density function

$$p(i,j) = Pr(NumberOfBoys = i, NumberOfGirls = j) = \frac{c}{1+i+j} \quad , \quad 0 \leq i \leq 3 \quad 0 \leq j \leq 3 \quad ,$$

for some constant c.

(i) Find c. (ii) Find the probability that a family has strictly more girls than boys.

(iii) Find the expected number of boys.

2. The joint density function of X and Y is given by

$$f(x,y) = \begin{cases} 6xy^2 \quad , \quad if \quad 0 < x < 1\,,\, 0 < y < 1; \\ 0 \quad otherwise \end{cases}$$

Find

(i) $P(X < \frac{1}{2}\,,\, Y > \frac{1}{2})$

(ii) $P(0 < X < 1\,,\, 0 < Y < 1)$

(iii) $P(0 < X < \frac{1}{4}\,,\, \frac{3}{4} < Y < 1)$

(iv) $E[X]$

(v) $E[Y]$

3. The return on two investments, X and Y, follows the joing density function

$$f(x,y) \;=\; \begin{cases} \frac{1}{2} \quad , \quad if \quad 0 < |x| + |y| < 1\,; \\ 0\,, \quad otherwise. \end{cases} \quad .$$

Find the marginal density functions $f_X(x)$ and $f_Y(y)$ and use them to find $Var(X)$ and $Var(Y)$.

4. A device runs until either of the two components fails, at which point the device stops running. The lifetimes of the two components has a joint probability density function

$$f(x,y) \;=\; \frac{x+y}{8} \quad , \quad for \quad 0 < x < 2 \quad and \quad 0 < y < 2 \quad .$$

What is the probability that the device fails during the first hour of operation?

5. Let X and Y be continuous random variables with joint density function

$$f(x,y) = \begin{cases} 15y \quad , & for \quad x^2 \le y \le x\,; \\ 0 \quad , & otherwise. \end{cases}$$

Let g be the marginal density function of Y. Find $g(y)$.

6. A company is reviewing tornado damage claims under a farm insurance policy. Let X be the portion of the claim representing damage to the house and let Y be the portion of the claim representing damage to the rest of the property. The joint density function of X and Y is

$$f(x,y) \;=\; \begin{cases} 6(1-x-y) \quad , & for \quad x>0\,,\, y>0\,,\, x+y<1 \\ 0 \quad otherwise. \end{cases}$$

Determine the probability that the portion of a claim representing damage to the house is less than the damage to the rest of the property.