

Dr. Z.'s Intro to Probability Homework assignment 11

1. The density function of a continuous random variable, X , is given by

$$f(x) = \begin{cases} \frac{x}{20} & \text{if } 3 \leq x \leq 7; \\ 0 & \text{otherwise.} \end{cases} .$$

(i) What is the probability that X is between 4 and 6?

(ii) What is the probability that X is between 3 and 5, if it is known that it is between 4 and 6.

2. Let X be the continuous random variable with density function

$$f(x) = \begin{cases} \frac{c}{x^2}, & 1 < x < 2; \\ 0 & , \text{ otherwise.} \end{cases}$$

for some constant c .

(a) Calculate $E[X]$, (b) Calculate $Var(X)$.

3. Let X be the continuous random variable with density function

$$f(x) = \begin{cases} \frac{|x|}{10}, & -2 < x < 4; \\ 0 & , \text{ otherwise.} \end{cases}$$

Calculate the expected value of X . Also calculate its variance

4. The lifetime of a machine part has continuous distribution on the interval $(0, 40)$, with probability density function f , where $f(x)$ is proportional to $(10 + x)^{-2}$. What is the probability that the lifetime of the machine part is less than 5 ?

5. The distribution of the size of claims paid under an insurance policy has probability density function

$$f(x) = \begin{cases} cx^a & , 0 < x < 5; \\ 0 & , \text{ otherwise.} \end{cases}$$

where $a > 0$ and $c > 0$.

For a randomly selected claim, the probability that the size of the claim is less than 3.75 is 0.487. Calculate the probability that the size of the claim is greater than 4.

6. Damage to a car in a crash is modeled by a random variable with density function

$$f(x) = \begin{cases} c(x^2 - 60x + 800) & , 0 < x < 20; \\ 0 & , \text{ otherwise.} \end{cases}$$

where c is a constant.

A particular car is insured with a deductible of 2. This car was involved in a crash with damage exceeding the deductible. Calculate the probability that the damage exceeded 10.

7. The lifetime in hours of a certain light bulb is given by

$$f(x) = \begin{cases} 0, & 0 < x < 100; \\ \frac{20000}{x^3}, & x > 100 \end{cases}$$

What is the probability that exactly 3 out of 6 light bulbs will have to be replaced within the first 200 hours?

8. The continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{1}{3}, & 0 < x < 1; \\ \frac{2}{5}, & 1 < x < 3; \\ 0 & \text{elsewhere.} \end{cases}$$

Let n be a positive integer, find $E[X^n]$.

9. The monthly profit of Company I can be modeled by a continuous random variable with density function f . Company II has a monthly profit that is three times that of Company I.

Determine the probability density function of the monthly profit of Company II.

10. A company agrees to accept the highest of four sealed bids on property. The four bids are regarded as four independent random variables with common cumulative distribution function

$$F(x) = x^3, \quad 0 < x < 1 \quad .$$

What is the expected value of the accepted bid?