

Dr. Z.'s Intro to Probability Homework assignment 1

1. A class has 10 boys and 5 girls. You have to pick a committee of 2 boys and 2 girls. In how many ways can you do it?
2. There are 20 athletes competing. One of them will get the Gold medal, another one the Silver medal, and yet another one the Bronze medal. In how many ways can it be done?
3. In how many ways can you rearrange the letters of (i) MISSISSIPPI (ii) NEWJERSEY (iii) AMERICA ?
4. In how many ways can you place 4 **identical** rooks on a chessboard in such a way that they don't attack each other?
5. Thirty items are arranged in a 6-by-5 array as shown

$$\begin{pmatrix} A_1 & A_2 & A_3 & A_4 & A_5 \\ A_6 & A_7 & A_8 & A_9 & A_{10} \\ A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\ A_{16} & A_{17} & A_{18} & A_{19} & A_{20} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} \\ A_{26} & A_{27} & A_{28} & A_{29} & A_{30} \end{pmatrix},$$

Calculate the number of ways to form a set of three distinct items such that no two of the selected items are in the same row or in the same column.

6. In how many ways can you place 5 X's and 4 O's in a three by three board, such that each cell gets either an X or an O? Out of these, how many of them have neither three X's nor three O's in any row or column?
7. By directly listing the set of 'successful elements' find the number of ways of
 - (i) Rolling three tetrahedral dice (each marked with 1, 2, 3, 4) so that their faces add-up to 10.
 - (ii) Rolling three 4-faced dice (each marked with 1, 2, 4, 8) so that their faces add-up to 20.
8. Recall that in Backgammon, you roll a pair of fair dice, and you move pieces accordingly to an available spot (or out of the board). Suppose that the game is about to finish and your opponent has one piece left one square away from the end, so if she makes it to the next turn (i.e. if you would be unable to go get out), she will win. Also recall that if you roll a double, then you get to move four times (e.g. a double 3 is the same as 3, 3, 3, 3).

For $1 \leq a \leq b \leq 6$, if right now you have a two pieces left, one a squares from the end and the other b squares from the end, what is your chance of winning?

Note: This problem has $\binom{7}{2} = 21$ parts. So be patient.