

## Answers to Dr. Z.'s Introduction to Probability homework assignments

**Disclaimer:** Not responsible for any errors. One dollar award to the **first** to find any error.

**Version of Dec. 4, 2017**

**Thanks:** 9/14/17: Riya Prabhudesai, who found an error in hw2#4 . 9/19/17: Rikab Gambhir, who corrected hw4#4. 9/26/17: TA Andy Huynh, who corrected hw5#7. 10/18/17: Ruizhe Fan, who corrected hw10#3. 10/27/17: Zhiyun Xu (who corrected hw13#1(c)(d)) 10/30/17: Thanks to Bennet Greenberg and Eric Mogen. 11/13/17: Aaron Kau (who corrected hw18#6). 11/15/17: Sining An (correcting hw16#6 [the former answer was the prob. that the poor husband makes *less* than 60K]). 11/20/17: Bennet Greenberg (correctiing the ordering of the answers to hw19#1). 11/25/17: Danielle Ferber (correcting hw12#6(c)). 12/4/17: Valerie Thaler (corrected hw20#5)  
**hw1:** **1.** 450; **2.** 6840 ; **3.** (i) 34650 (ii) 60480 (iii) 2520; **4.** 117600 ; **5.** 1200 ; **6.** 126, 45 ; **7.** (i) 6 ; (ii) 3 ; **8.** According to  $(a, b)$  being

$$[[1, 1], [1, 2], [1, 3], [1, 4], [1, 5], [1, 6], [2, 2], [2, 3], [2, 4], [2, 5], [2, 6], [3, 3], [3, 4], [3, 5], [3, 6], [4, 4], [4, 5], [4, 6], [5, 5], [5, 6], [6, 6]] \quad ,$$

The probabilities, respectively, are

$$[1, 1, \frac{17}{18}, \frac{29}{36}, \frac{23}{36}, \frac{5}{12}, \frac{13}{18}, \frac{25}{36}, \frac{23}{36}, \frac{19}{36}, \frac{13}{36}, \frac{17}{36}, \frac{17}{36}, \frac{7}{18}, \frac{5}{18}, \frac{11}{36}, \frac{5}{18}, 2/9, 1/6, 1/6, 1/9] \quad .$$

**hw2:** **1.**  $\emptyset, \{7, 8\}, \{1\}, \{5\}, \{2, 4, 6\}, \{1, 3, 5, 7, 8\}$  ; **2.** (i) 0.7 (ii) 0.4 (iii) 0; **3.** 3 ; **4.** 57 ; **5.** 0.48 ; **6.** 15 ; **7.** 30, 25 .

**hw3:** **1.**  $\binom{n}{k}/2^n$  ; **2.**  $\frac{625}{1296}$  ; **3.**  $\frac{1}{2^n}$  ; **4.**  $\frac{63}{46189} = 0.00136396111\dots$  ; **5.** 0.7063162427... ; **6.** 0.7771990741... ; **7.** 0.0750546579....

**hw4:** **1.**  $\frac{1}{2}, \frac{1}{2}, \frac{1}{7}, \frac{1}{4}, 1$  ; **2.** 0.0075 ; **3.** 0.03 ; **4.** 0.6, 0.4, 0.2 ; **5.**  $\frac{7}{15}$  ; **6.**  $20/1019 = 0.01962708538\dots$  ; **7.** 0.2191780822, 0.1643835616, 0.5753424658, 0.04109589041; **8.** 0.2922077922... ; **9.**  $\frac{2}{5}$  .

**hw5:** **1.** 0.0282475249... , 0.1210608210... ; **2.** 0.04824344307... ; **3.**  $\frac{1}{8}$  ; **4.** 0.0058060800... ; **5.**  $\frac{3}{5}, \frac{3}{10}, \frac{1}{10}$  ; **6.** 0.5327128713... ; **7.**  $\frac{4528}{15625} = 0.289792$  .

**hw6:** **1.** (a)  $\frac{1}{36}, \frac{1}{18}, \frac{1}{18}, \frac{1}{12}, \frac{1}{18}$ ; (b)  $\frac{49}{4}$  ; **2.** (a) 39/55, 3/11, 1/55; (b) 0; **3.** (a) 60/137, 30/137, 20/137, 15/137, 12/137; (b)  $300/137 = 2.189781022\dots$  ; **4.** (a)  $\frac{n+1}{2}$ ; (b)  $\frac{(n+1)(2n+1)}{6}$ ; (c)  $\frac{n(n+1)^2}{4}$  ; **5.** 2; **6.**  $\frac{2}{5} = 0.4$  .

**hw7:** **1.** (a) 3.3; (b) 0.6443842677... ; **2.** 0 ; **3.** (a) 5.1; (b) 6.89, 2.624880950... ; **4.**  $\frac{72}{55} = 1.309090909\dots, 1.144155107\dots$  ; **5.** (a) 60/137, 30/137, 20/137, 15/137, 12/137; (b) 1.774202142... , 1.331991795... ; **6.**  $\frac{(n-1)(n+1)}{12}, \frac{\sqrt{(n-1)(n+1)}}{2\sqrt{3}}$  ; **7.** 56000, 77871.68934... ; **8.** 235.4654964....

**hw8:** **1.** 0.3222004736 ; **2.**  $\frac{1}{2^n} \sum_{k=a+1}^{b-1} \binom{n}{k}$  ; **3.** 0.8450192681917... ; **4.** 0.39995646... ; **5.** 0.038533403939... ; **6.** 70, 4.58257569495584... ; **7.** 0.19104443929....

**hw9:** **1.** 0.2231301601..., 0.191153169...; **2.** 0.6115846827...; **3.** (i) 0.14936120..., 0.141303860...; (ii) 0.07326255555, 0.06569281...; **4.** 0.401286164...; **5.** 0.57680991887...; **6.** 2; **7.**  $4/e^3 = 0.199148273\dots$

**hw10:** **1.** 5, 4.472135955...; **2.** 0.5248, 5,  $\frac{10}{3}$ ; **3.**  $\binom{n+9}{9}/2^{n+10}$  [corrected 10/18/17 thanks to Ruizhe Fan]; **4.** 0.5223047494..., 0.8909904548...; **5.**  $150p$ ; **6.** 30.

**hw11:** **1.**  $\frac{1}{2}, \frac{9}{20}$ ; **2.**  $2\ln(2) = 1.386294361\dots, 2 - 4\ln(2)^2 = 0.078187944\dots$ ; **3.**  $\frac{28}{15} = 1.86666666\dots, \frac{746}{225} = 3.315555556\dots$ ; **4.**  $\frac{5}{12}$ ; **5.** 0.4276935604; **6.**  $\frac{125}{486} = 0.2572016461\dots$ ; **7.**  $\frac{135}{1024} = 0.1318359375\dots$ ; **8.**  $\frac{6 \cdot 3^n - 1}{5(n+1)}$ ; **9.**  $\frac{1}{3}f\left(\frac{x}{3}\right)$ ; **10.**  $\frac{12}{13}$ .

**hw12:** **1.**  $\frac{4}{15}, \frac{2}{5}, \frac{2}{5}$ ; **2.**  $\frac{A}{2}, \frac{A^2}{12}$ ; **3.**  $\frac{1}{5}, \frac{7}{15}, \frac{1}{3}$ ; **4.**  $\frac{27\sqrt{13}}{100} = 0.9734988442\dots$ ; **5.** 0.3780661223..., 0.9087887803..., 0.3173105079; **6.** 0.1308, 0.0629, 0.6702344537...; **7.** 2.31686...

**hw13:** **1.** 0.04966365857..., 0.000006144212353,  $\frac{1}{3}, \frac{1}{9}$ ; **2.** 0.6703200460...; **3.** 0.1353352832...; **4.** 10255.899...; **5.** 1.048891322; **6.** 5644.226968...

**hw14:** **1.**  $\frac{105}{533} = 0.1969981238\dots, \frac{357}{1066} = 0.33489681\dots, \frac{1147}{1066} = 1.075984991\dots$ ; **2.**  $\frac{7}{32} = 0.21875$ ,  $1, \frac{37}{1024} = 0.03613281250, \frac{2}{3}, \frac{3}{4}$ ; **3.**  $1 - |x|$  ( $-1 \leq x \leq 1$ ),  $1 - |y|$  ( $-1 \leq y \leq 1$ ),  $\frac{1}{6}, \frac{1}{6}$ ; **4.**  $\frac{5}{8} = 0.625$ ; **5.**  $15y(\sqrt{y} - y)$ ; **6.**  $\frac{1}{2}$ .

**hw15:** **1.**  $\frac{9}{16} = 0.5625$ ; **2.**  $\frac{128}{243} = 0.5267489712\dots$ ; **4.**  $\frac{10}{3}(e^{-2z} - e^{-5z})$ ; **5.** 0.1562934519...; **6.** 2.5, 1.837117307...; **7.**  $e^{-\sum_{i=1}^n \mu_i} (\sum_{i=1}^n \mu_i)^3 / 6$

**hw16:** **1.**  $\frac{3600}{769} (i+3)^{-2}$  if ( $0 \leq i \leq 2$ ), 0 otherwise. OR(explicitly):  $p(0|2) = 400/769, p(1|2) = 225/769, p(2|2) = 144/769$ ; **2.**  $\frac{11}{15}$ ; **3.**  $2 \frac{x+y}{1+2y}, \frac{69}{220} = 0.3136363636\dots$ ; **4.**  $\frac{2(x+y)}{1-y^2}, \frac{2(x+y)}{1-x^2}, \frac{8}{15} = 0.533333333\dots$ ; **5.**  $\frac{5}{12} = 0.41666666\dots$ ; **6.** 0.2767857142...

**hw17:** **2.**  $\frac{n-4}{32}, \frac{n-1}{4}$ ; **3.**  $np(1-p)$ ; **4.**  $\frac{7}{12}, \frac{7}{12}, \frac{7}{6}, \frac{5}{36}$ ; **5.**  $0, \frac{1}{3}$ ; **6.**  $\frac{5}{36}$ ; **7.**  $\frac{5}{12}$ ; **8.**  $\frac{7381}{252} = 29.28968254\dots$

**hw18:** **1.**  $(0.01x^{100} + 0.99x^{-1})^n, n/100, 100.9899n$ ; **2.** [1, 2, 2, 3, 3, 4], [1, 3, 4, 5, 6, 8]; **3.**  $(\frac{3}{4}x + \frac{1}{8}x^2 + \frac{1}{8}x^3)^n, \frac{11n}{8}, \frac{31n}{64}$ ; **6.**  $1 - 3.727075259 \times 10^{-11} = 0.99999999996272924741\dots$  (practically certain to be a loser).

**hw19:** **1.** 0.6546536709..., 0.6546536709..., 1; **2.** -1; **3.** (a) 2,2; (b)  $\frac{2}{3}, \frac{2}{3}$ ; (c)  $-\frac{2}{3}, -1$ ; **4.** 2800, 200; **5.** 9, 8; **6.** 19300; **7.**  $\Phi(1) = 0.8413447461\dots$

**hw20:**

**1.**

(a) factor(add(j/(1+i+j), j=0..3)/add(1/(i+j+1), j=0..3)); ( $0 \leq i \leq 3$ )

This yields: 23/25, 86/77, 23/19, 404/319 for  $i = 0, i = 1, i = 2, i = 3$  respectively.

(b) `factor(add(i/(1+i+j),i=0..3)/add(1/(i+j+1),i=0..3));`

This yields:  $23/25, 86/77, 23/19, 404/319$  for  $j = 0, j = 1, j = 2, j = 3$  respectively.

(c) `add(add(j/(1+i+j),j=i..3),i=0..3)/add(add(1/(1+i+j),j=i..3),i=0..3);`

This yields  $\frac{1965}{1418} = 1.385754584\dots$

(d) `add(add(i/(1+i+j),j=i..3),i=0..3)/add(add(1/(1+i+j),j=i..3),i=0..3);`

This yields  $\frac{817}{1418} = 0.5761636107\dots$

**2.** (a)  $\frac{3y+2}{3(2y+1)}$  ( $0 \leq y \leq 1$ ) (otherwise 0) (b)  $\frac{3x+2}{3(2x+1)}$  ( $0 \leq x \leq 1$ ) (otherwise 0)

**3.** (a) `factor(int(x*12*(x**2+y)/5,x=y..1)/int(12*(x**2+y)/5,x=y..1));`

This yields  $\frac{3(y+1)^3}{4(y^2+4y+1)}$

(b) `factor(int(y*12*(x**2+y)/5,y=0..x)/int(12*(x**2+y)/5,y=0..x));`. This yields  $\frac{x(2+3x)}{3(2x+1)}$ .

**4.** (a) `int(int(12/5*x*(x**2+y),x=y..1-y),y=0..1/2)/ int(int(12/5*(x**2+y),x=y..1-y),y=0..1/2);`

This equals  $\frac{13}{22}$ .

(b) `int(int(12/5*y*(x**2+y),x=y..1-y),y=0..1/2)/ int(int(12/5*(x**2+y),x=y..1-y),y=0..1/2);`

This equals  $\frac{21}{110}$ .

**5.** 896.9072165... ; **6.** 14.5 hours .

**hw21 1.**  $100/(1 + 2 \ln 2) = 41.90597842$  ; **2.** 8,80,16; **3.** 5000 ; **4.**  $\frac{1}{2}$ ; **5.** 10560; **6.**  $e^{t_1^2+t_2^2}$  .

**hw22 2.**  $0.195767\dots < 0.857142$  ; **4.**  $\frac{4}{5}$ , it is  $\geq \frac{23}{48}$  ; **5.**  $\frac{9}{7}$ ,  $\frac{\sqrt{3}}{7}$ ,  $0.059586024 < \frac{1}{4}$  ; **6.** (a) it is  $\leq \frac{10}{11}$  ; (b) it is  $\geq \frac{3}{4}$  .

**hw23 2.** 384, 2000 ; **3.**  $\Phi(1.6035) - \Phi(-4.8107) = 0.94559$  ; **4.** (i)  $25, \frac{25}{3}$  ; (ii)  $\Phi(\frac{2\sqrt{3}}{5}) - \Phi(\frac{-4\sqrt{3}}{5}) = 0.6312678473\dots$  **5.**  $\Phi(2\sqrt{3}/3) = 0.8758934605\dots$  .