NAME: (print!) Alexandra Feldler

E-Mail address: arf98@scarletmail.rutgers.edu

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

continued from #5.

We have only two possibilities $a_iH \cap a_jH = \emptyset$ or $a_iH = a_jH$. Moreover from lemma α [If h_1 and h_2 are 2 distinct elements from H then ah_1 and ah_2 are also distinct, since otherwise $ah_1 = ah_2 \Rightarrow a^{-1}ah_1 = a^{-1}ah_2 \Rightarrow h_1 = h_2$, which is a contradiction. So if we multiply all elements of H by a, we obtain the same H elements which means that $|aH| = |H|$]. It follows that all cosets have exactly $|H|$ number of elements. Therefore $|G| = |H| + |H| + \dots + |H| \Rightarrow |G| = a|H|$ and $a = \frac{|G|}{|H|}$ is always an integer. QED.

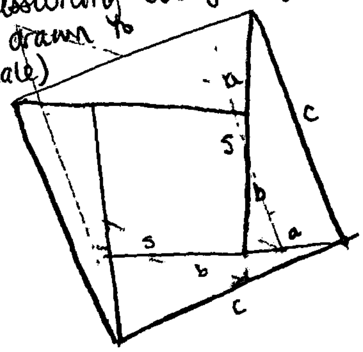
1. 10 (out of 10)
2. 5 (out of 10)
3. 7 (out of 10)
4. 6 (out of 10)
5. 10 (out of 10)
6. 10 (out of 10)
7. 0 (out of 10)
8. 0 (out of 10)
9. 0 (out of 10)
10. 0 (out of 10)
11. 10 (out of 10)

total: (out of 110)

58 · 1.134 = 66

1. (10 pts.) Give two proofs of the Pythagorean theorem.

Proof 1: (assuming everything is drawn to scale)



Total area = hypotenuse squared

inscribed area = four triangles + one square

$s = b - a$

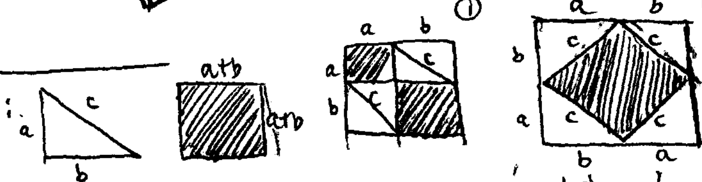
$c^2 = 4 \cdot \frac{ab}{2} + s^2$

$c^2 = 2ab + (b-a)^2 = 2ab + b^2 - 2ab + a^2 = b^2 + a^2$

$\Rightarrow c^2 = a^2 + b^2$. QED.

10

Proof 2:



Suppose that you start w/ 4 right triangles w/ sides of length a, b, and c and a square tray w/ 4 sides of length a+b. You can arrange the triangles into the tray in 2 different ways as shown above. In ① you leave 2 square holes. In ② you leave one large square hole. This has an area of c^2. Since they are equal. $a^2 + b^2 = c^2$. QED.

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

Assume that $\sqrt[7]{3}$ is rational.

[lemma: every positive integer N can be written as $N = 3^i a$ where $\gcd(a,3) = 1$ for some integer $i \geq 0$]

Assume $\sqrt[7]{3} = \frac{m}{n}$, where m, n are some rational positive integers.

So, $3 = \frac{m^7}{n^7} \Rightarrow 3n^7 = m^7$. ($\alpha$)

Say $n = 3^j b$ and $m = 3^i a$, for some positive integers i and j.

Plugging into α: $3(3^j b)^7 = (3^i a)^7$

$3^{7j+1} b = 3^{7i} a$.

Wrong

But $7j+1$ is odd and $7i$ is even and we assumed they are equal. Contradiction. Therefore $\sqrt[7]{3}$ is irrational. QED

3. ~~(10 pts. total) (a) (5 points) Define the Mandelbrot set.~~

The Mandelbrot set is the set M that consists of all those complex c-values for which the corresponding orbit of 0 under $x^2 + c$ does not escape to infinity.

c could include 0, -1, and i but does not include 1 and 2.

5

(b) (5 points) Define the Feigenbaum constant. Explain everything!

$$\delta = \lim_{n\to\infty} \frac{a_{n-1} - a_{n-2}}{a_n - a_{n-1}} = 4.669201609102\ldots$$

What is a?

2

4. (10 pts. total)
(a) (6 pts.) For any position P in the (n^2-1)-puzzle, let $[i, j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1, 2, 3, \ldots, n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + \ inv\,(\pi(P)) \quad ,$$

where $inv\,(\pi)$ is the number of inversions of the permutation π.
Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd.**
Note: You may use the lemma that if you exchange *any* two elements in a permutation, the number of inversions *always* changes by an odd integer.

$S(P_0) \equiv S(P_{n-1}) \mod 2$

$S(P')$ has the same parity as $S(P)$ $[S(P) = inv(\pi) + i + j]$
for $P_0, P_1, P_2, \ldots, P_n$.

$S(P_n) \equiv S(P_{n-1}) \mod 2$

By the lemma,

why?

2

(b) (4 pts)
Let

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & \\ 6 & 7 & 8 \end{pmatrix} \quad .$$

Can you reach position ~~P~~ Q from position P, by a sequence of legal moves? Explain!

$P = 123594678$

$inv(P) = 0+0+0+1+4+0+0+0+0$
$= 5$

$i = 2, j = 2$

$\pi = 5 + 2 + 2 = 9.$

$Q = 421359678$

$inv(Q) = 3+1+0+0+3+0+0+0 = 7$

$i = 2 \quad j = 3$

$\pi = 7 + 2 + 3 = 12$

It is impossible bc $\pi(P)$ is odd and $\pi(Q)$ is even.

QED.

4

10

5. (10 points)
Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

Let $|G| = t$ and $\{a_1H, a_2H, \ldots, a_iH\}$ be the family of all cosets of H in G. Then $G = a_1H \cup a_2H \cup \ldots \cup a_iH$ because $G = \{a_1, a_2, \ldots, a_t\}$ and $1 \in H$. By the lemma [let $aH \cap bH \neq \emptyset$, then there exists an element x w/ $x \in aH \cap bH$ $\Rightarrow ah_1 = x = bh_2 \Rightarrow b^{-1}a = h_2h_1^{-1} \in H$, $\therefore aH = bH$], for any two cosets a_iH and a_jH $\rightarrow$

mile

continue onto front

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x,y) + iv(x,y)$ where $u(x,y), v(x,y)$ satisfy the above system of two equations?

Cauchy - Riemann Equation.

$u + iv = F(x+iy)$, where F is analytic

10

7. (10 points) Who discovered the quaternions? What city did that person live in?

0

8. (10 points) What is Heron's formula, what century did Heron live in?

early 14th century

0

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

0

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

Ireland

0

11. (10 points total)
(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$\cos\frac{\pi}{4} \cos\frac{\pi}{8} \cos\frac{\pi}{16} \cos\frac{\pi}{32} \cdots$

10

(b) (5 points) State the names of two people who initiated the use of logarithms

John Napier and Briggs

/

NAME: (print!) CASSANDRA SANCHEZ

E-Mail address: cms548@scarletmail.rutgers.edu

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

1. 10 (out of 10)
2. 9 (out of 10)
3. 4 (out of 10)
4. 8 (out of 10)
5. 6 (out of 10)
6. 10 (out of 10)
7. 10 (out of 10)
8. 10 (out of 10)
9. 10 (out of 10)
10. 10 (out of 10)
11. 10 (out of 10)

total: (out of 110)

$97 \times 1.134 = \underline{110}$

5. Let $|H| = m$, $|G| = n$, $H \subset G$. Then $\exists$ an element $g_2 \in G$ st $g_2 \notin H$. Let $g_2H = \{g_2h_1, \dots, g_2h_m\}$

Lemma: $g_2h_i = g_2h_j$ then $g_2^{-1}(g_2h_i) = g_2^{-1}(g_2h_j)$ but by associativity, $(g_2^{-1}g)h_i = (g_2^{-1}g)h_j$ so $eh_i = eh_j \Rightarrow h_i = h_j$. So $g_2h_i \neq g_2h_j$ unless $i=j$, which is trivial.

Lemma: Suppose $g_2h_i = h_j$ then $g_2h_i(h_i^{-1}) = h_jh_i^{-1}$ and $g_2 = h_jh_i^{-1}$ but $g \notin H \rightarrow$ Contradiction so $g_2H \cap H = \emptyset$

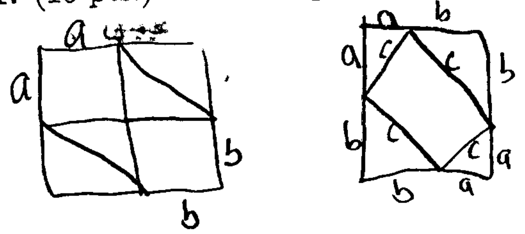
So we can build a set and repeat for $g \notin H$. Repeating this until we are done, say r times, we see $n = rm \Rightarrow r = \frac{n}{m} = \frac{|G|}{|H|}$ so $\frac{|G|}{|H|}$ is always an integer ∎

not clear
what are the lemmas?

(10)

1. (10 pts.) Give two proofs of the Pythagorean theorem.

Proof 1:



both Squares have area $(a+b)^2$

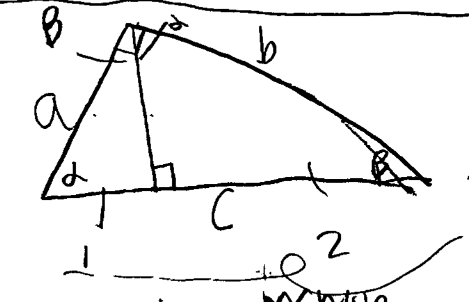
Let $4T$ = Area of the 4 triangles

$(a+b)^2 = a^2+b^2+\cancel{4T = c^2+4T}$

Area $= a^2+b^2+4T$

Area $= c^2+4T \Rightarrow$ so $a^2+b^2=c^2$ ∎

Proof 2:



$\alpha+\beta=90$ $\frac{1}{2}bh$

∴ Area of Whole triangle = Area of $\Delta_1 + \Delta_2$

There is a lemma about ratios of triangles that states there is a constant k.

By the lemma, we know that $kc^2 = ka^2 + kb^2 \Rightarrow c^2 = a^2+b^2$ ∎

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

Proof: Assume $\sqrt[7]{3}$ is rational. Then $\sqrt[7]{3} = \frac{m}{n}$. Then by algebra we see $(\sqrt[7]{3})^7 = \left(\frac{m}{n}\right)^7 \Rightarrow 3 = \frac{m^7}{n^7} \Rightarrow 3n^7 = m^7$. Because we can express any integer N as its prime factorization we have $N = 3^i a$ for $i \geq 0$ and $\gcd(a,3)=1$. So we can write $m = 3^i \bar{m}$ for $i \geq 0$, $\gcd(\bar{m},3)=1$ and $n = 3^j \bar{n}$ w/ $j \geq 0$, $(\bar{n},3)=1$. Then $3(3^j\bar{n})^7 = (3^i\bar{m})^7 \Rightarrow$ $3^{7j+1}\bar{n}^7 = 3^{7i}\bar{m}^7$ Since neither $\bar{n}$ or $\bar{m}$ is divisible by 7 we see $7j+1 = 7i$ which is impossible. Why?

So $\sqrt[7]{3}$ is irrational ∎

(9)

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

$$Z \to Z^2 + C$$

explain!

(2)

(b) (5 points) Define the Feigenbaum constant. Explain everything!

what is r_n?

$$\lim_{n\to\infty}\left(\frac{r_n - r_{n-1}}{r_{n+1} - r_n}\right) = 4.669\ldots$$

(2)

4. (10 pts. total)

(a) (6 pts.) For any position P in the $(n^2$-1)-puzzle, let $[i, j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1, 2, 3, \ldots, n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + inv(\pi(P)) \quad ,$$

where $inv(\pi)$ is the number of inversions of the permutation π.

Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd**.

Note: You may use the lemma that if you exchange *any* two elements in a permutation, the number of inversions *always* changes by an odd integer.

4

Proof By Induction: Assume (★) to be true: not clear

Base Case: If there is only one inversion then you are only exchanging two elements so the number of inversions changes by ± odd, and i or j changes by ±1

Inductive Step: Let Q be the new position obtained by a finite number of moves, and the moves from P to the Q−1st position have not changed the parity. Then by the lemma, one more move to Q changes the number of inversions by an odd integer and $i' = i \pm 1$ and $j' = j$ or $j' = j \pm 1$, $i' = i$ which is an odd change so odd+odd = even ∴ the parity is the same

(b) (4 pts) Let

4

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & \\ 6 & 7 & 8 \end{pmatrix}$$

Can you reach position Q from position P, by a sequence of legal moves? Explain!

NO | P = 1 2 3 5 9 4 6 7 8 (0)(0)(0)(1)(4)(0)(0)(0)(0) Inv(P) = 5 i = 2 j = 2 So S = 5+2+2 = 9

Q = 4 2 1 3 5 9 6 7 8 (3, (1, (0) (0) (0) (3) (0)(0) Inv(Q) = 3+1+3 = 6+1 = 7 i = 2, j = 3

S = 7+2+3 = 12

It is impossible because 9 is odd but 12 is even, and by part (a) we know this is $\boxed{\text{impossible}}$

5. (10 points)

Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

6

On Front

6. (10 points) What is the name of the following famous equation-pair?

$$\boxed{u_x = v_y \quad , \quad u_y = -v_x}$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $\boxed{u(x,y) + iv(x,y)}$ where $u(x,y), v(x,y)$ satisfy the above system of two equations?

- Name: Cauchy - Riemann Equations
- The function that satisfies this is special because it is a complex function, analytic

10

7. (10 points) Who discovered the quaternions? What city did that person live in?

- Hamilton — Dublin

10

8. (10 points) What is Heron's formula, what century did Heron live in?

~62 AD →

Area of Δ: $A = \sqrt{s(s-a)(s-b)(s-c)}$ 1^{st} Century AD

10

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

- Cambridge
- Issac Barrow
- Barrow gave Newton his professorship position
- Warden/master of the mint

10

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

- Leipzig
- most life near court of Hanover
- Employed by King George

10

11. (10 points total)

(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$$\frac{2}{\pi} = \cos\frac{\pi}{4} \cos\frac{\pi}{8} \cos\frac{\pi}{16} \cos\frac{\pi}{32} \cdots$$

5

(b) (5 points) State the names of two people who initiated the use of logarithms

(1) John Napier (ana John Neper)

(2) Briggs

5

/

NAME: (print!) Shmuel Lotsvin

E-Mail address: slotsvin@gmail.com

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

1. 10 (out of 10)
2. 6 (out of 10)
3. 7 (out of 10)
4. 8 (out of 10)
5. 8 (out of 10)
6. 5 (out of 10)
7. 10 (out of 10)
8. 10 (out of 10)
9. 10 (out of 10)
10. 10 (out of 10)
11. 10 (out of 10)

total: 94 (out of 110) $\times 1.134 =$

106.6

5) Let H be a subgroup of group G, and let the order of H be $|H|$ and the order of G be $|G|$. We want to show $|G|/|H| = n$ for some $n \in \mathbb{Z}^+$. Pick an $r_1 \in G$. Then, $|Hr_1| = |H|$ *. If $Hr_1 = G$, then $|G|/|H| = 1 = n$. Otherwise, we choose $r_2 \in G \setminus Hr_1$. By another lemma, we know that $Hr_1 \cap Hr_2 = \emptyset$, so they are disjoint and $|Hr_1 \cup Hr_2| = 2|H|$. If $H_1 \cup Hr_2 = G$, we are done and $|G|/|H| = 2 = n$. Otherwise, we continue in this way, picking elements from G that are not in any cosets we have already made, until we reach $Hr_1 \cup Hr_2 \cup \ldots \cup Hr_n = G$. Then, $|G| = n|H|$. So, we will eventually have $\frac{|G|}{|H|} = n$ for some $n \in \mathbb{Z}^+$, as desired.

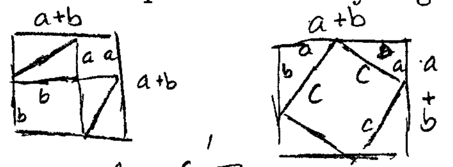
8

Prove the lemmas! You had to prove

* Note: By lemma, no elements in Hr_1 should be equal, i.e, for $x_i, x_j \in Hr_1$, $i \neq j$, we have $x_i \neq x_j$. So, $|Hr_1| = |H|$. This true for Hr_i for $i = 1, \ldots, n$.

1. (10 pts.) Give **two** proofs of the Pythagorean theorem.

Proof 1:



Area of □s : $(a+b)^2 = (a+b)^2$

So, we have $4(\frac{1}{2}ab) + a^2+b^2 = 4(\frac{1}{2}ab) + c^2$

$\Leftrightarrow \quad a^2+b^2 = c^2$

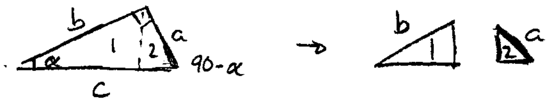
So, $a^2+b^2=c^2$, as desired.

(Note: $4(\frac{1}{2}ab)$ describes the triangles)

(Note: $(a+b)^2 = a^2+2ab+b^2 = a^2+(4\cdot\frac{1}{2}ab)+b^2$)

10

Proof 2:



We have similar triangles 1, 2, and the entire triangle (3). By lemma, we know that a family of similar triangles have a similar area within a constant k. So:

$(\text{Area of } 1)\cdot k + (\text{Area of } 2)\cdot k = (\text{Area of } 3)\cdot k$

$(b^2+a^2)k = c^2 k \iff a^2+b^2=c^2$, as desired.

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

Suppose for contradiction that $\sqrt[7]{3}$ is rational. Then, $\sqrt[7]{3} = m/n$ for some $m,n \in \mathbb{N}$.

So, we get: $\sqrt[7]{3} = m/n \iff 3 = m^7/n^7 \iff m^7 = 3n^7$

By lemma, we can write m as the product $3^i a$, where $\gcd(3,a)=1$ and $i,a \in \mathbb{N}$.

wrong Similarly, we can write n as $3^j b$, where $\gcd(3,b)=1$ and $j,b \in \mathbb{N}$

So, we get $m^7 = 3^{7i}a^7$ and $n^7 = 3^{7j}b^7$. So, $m = 3^{7i}a^7 = 3^{7j+1}b^7$. But, this cannot happen, as that means an odd and an even number of factors of 3 can be factored from m (that is, $7i \neq 7j+1$). So, we have a contradiction, so $\sqrt[3]{7}$ is irrational.

6

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

Mandelbrot Set M = a set of values c in the complex plane that orbit around 0 (i.e. $z=0$) such that $z \to z^2+c$, and is bounded. For example, $c=i \in M$ but $c=1 \notin M$

5

Note: Bounded refers to the sequence of $z_0, z_1, z_2, \ldots$, created by applying z_0 and $z_{n+1} = z_n + c$, for some c.

(b) (5 points) Define the Feigenbaum constant. Explain everything!

$$\lim_{n\to\infty} \frac{r_n - r_{n-1}}{r_{n+1} - r_n} = 4.669$$

2 what is r_n?

4. (10 pts. total)
(a) (6 pts.) For any position P in the $(n^2$-1$)$-puzzle, let $[i, j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1, 2, 3, \ldots, n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + \, inv\,(\pi(P)) \quad ,$$

where $inv\,(\pi)$ is the number of inversions of the permutation π.
Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd**.
Note: You may use the lemma that if you exchange *any* **two** elements in a permutation, the number of inversions *always* changes by an odd integer.

Suppose Q is reachable from P in a finite number of legal moves. Each legal move changes the parity of P, S(P), by an even number; this is because:
- moving the n^2 tile changes the parity of the number of inversions by an odd number
- the i and j are now inverted, changing parity by +1 or −1.
- odd + 1 = even, odd − 1 = even, so the moves always change parity by even number.

& S(P)

4

Since an odd parity cannot be reached, if S(P) = even, through an finite number of legal moves, as even + even = even; and since an even parity cannot be reached, if S(P) = odd, from a finite number of legal moves, we must have that S(P) = S(Q). So, they are both even or both odd, as desired.

(b) (4 pts)
Let

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & 9 & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & 9 \\ 6 & 7 & 8 \end{pmatrix} \quad .$$

Can you reach position ~~P~~ Q from position P, by a sequence of legal moves? Explain!

S(P) = (1+4) + 2+2 = 9 , S(Q) = (3+1+3) + 3+2 = 12

4

Q is not reachable from P through a series of legal moves because the parity of P, S(P), is odd while the parity of Q, S(Q), is even. Since every legal move changes the parity of the position by an even amount, S(P) will be odd within a finite number of moves and will not reach S(Q).

5. (10 points)
Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x,y) + iv(x,y)$ where $u(x,y), v(x,y)$ satisfy the above system of two equations?

- Cauchy-Riemann Equations ✓

- it is equivalent to $\cos(\Theta) + i\sin(\Theta)$ ✗

5

7. (10 points) Who discovered the quaternions? What city did that person live in?

- William Rowan Hamilton
- Dublin ✓

10

8. (10 points) What is Heron's formula, what century did Heron live in?

- 1^{st} century ✓
- Area of a triangle : $\sqrt{S(S-a)(S-b)(S-c)}$ ✓

10

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

- Cambridge ✓
- Barrow ✓
- Gave Newton his own professor position ✓
- Warden of the mint? ✓

10

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

- Leipzig ✓
- Hannover? ✓
- King George : I? ✓

10

11. (10 points total)
(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$2/\pi = \cos(\pi/4)\cos(\pi/8)\cos(\pi/16)\cos(\pi/32)\ldots$ ✓

10

(b) (5 points) State the names of two people who initiated the use of logarithms

- John Nepper
- Henry Briggs ✓

/

NAME: (print!) Sarah Teklinski

E-Mail address: sarah.teklinski@rutgers.edu

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTE-BOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

1. 10 (out of 10)
2. 10 (out of 10)
3. 7 (out of 10)
4. 10 (out of 10)
5. 7 (out of 10)
6. 10 (out of 10)
7. 10 (out of 10)
8. 10 (out of 10)
9. 6 (out of 10)
10. 0 (out of 10)
11. 10 (out of 10)

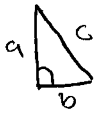
total: (out of 110)

$90 \times 1.134 = 102.1$

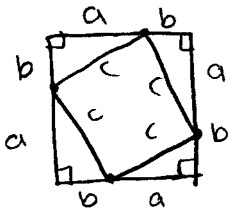
5 (cont): $g_k H = \{g_k h_1, g_k h_2, \ldots, g_k h_m\}$. There are k cosets total and m elements in each coset of H. Since G is the disjoint union of all k cosets, $n = mk$. Thus $|G| = |H|k$, $\Rightarrow \frac{|G|}{|H|} = k$, which is always an integer. ∎

Why? You have to prove it

1. (10 pts.) Give two proofs of the Pythagorean theorem.

Pf 1: Consider the triangle  . Now consider the square of side length $a+b$:

10

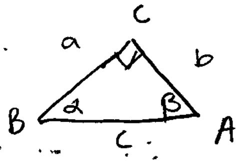
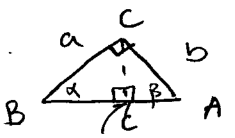


The area of the square is $(a+b)^2$. The area also equals the area of the 4 triangles plus the area of square with side length c.

Thus $(a+b)^2 = 4(\frac{1}{2}ab) + c^2$

$a^2+2ab+b^2 = 2ab+c^2$

$a^2+b^2 = c^2$ ■

Pf 2: Consider the triangle  Draw an altitude onto side length c:  . We know triangles ABC, BCD, and ACD are similar. Thus from the lemma, for some $k \in \mathbb{Z}^+$, $kc^2 = ka^2 + kb^2$.

Thus $a^2+b^2 = c^2$. ■

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

Pf: Suppose not. Then for $m, n \in \mathbb{Z}$, m, n coprime, $\sqrt[7]{3} = \frac{m}{n} \Rightarrow 3 = \frac{m^7}{n^7} \Rightarrow m^7 = 3n^7$. Thus m^7 is divisible by 3 $\Rightarrow m$ is divisible by 3. Thus for $p \in \mathbb{Z}^+$, $m = 3p$, $(3p)^7 = 3n^7 \Rightarrow n^7 = 3^6 p^7$.

Thus n^7 is divisible by 3 $\Rightarrow n$ is divisible by 3. We have arrived at a contradiction since m, n must be coprime but both m, n are divisible by 3. Thus $\sqrt[7]{3}$ is irrational. ■

6

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

The Mandelbrot set M consists of all complex numbers c such that $z \to z^2+c$, where the iterations of $z \to z^2+c$, $(z^2+c) \to (z^2+c)^2+c$, $(z^2+c)^2+c \to ((z^2+c)^2+c)^2+c$, ... do not get infinitely large

5

(b) (5 points) Define the Feigenbaum constant. Explain everything!

From the formula $x_{n+1} = r x_n (1-x_n)$, the Feigenbaum constant is $\lim_{n\to\infty} \frac{r_n - r_{n-1}}{r_{n+1} - r_n} \approx 4.669\ldots$

what is r_n?

2

4. (10 pts. total)

(a) (6 pts.) For any position P in the (n^2-1)-puzzle, let $[i, j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1, 2, 3, \ldots, n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + \mathit{inv}\,(\pi(P)) \quad ,$$

where $\mathit{inv}\,(\pi)$ is the number of inversions of the permutation π.
Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd**.
Note: You may use the lemma that if you exchange *any* **two** elements in a permutation, the number of inversions *always* changes by an odd integer.

6

Pf: In any legal move, we exchange the blank space with a neighboring space. So either i increases by 1 or decreases by 1 or j increases by 1 or decreases by 1. So $i+j$ changes by ± 1. From the lemma, if we exchange any 2 elements in a permutation the number of inversions always changes by an odd integer. So $inv(S(G)) = 2p+1$ for $p \in \mathbb{Z}$, thus the next move will have a parity of even, since odd ± 1 is an even integer. If the parity of $S(P)$ is even, since even + even is even, $S(Q)$ will have even parity. If the parity of $S(P)$ is odd, since odd + even is odd, $S(Q)$ will have odd parity. Thus in either case, parity of $S(P)$ equals parity of $S(Q)$.

4

(b) (4 pts) Let

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & \boxed{9} & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & \boxed{9} \\ 6 & 7 & 8 \end{pmatrix}$$

Can you reach position Q from position P, by a sequence of legal moves? Explain!

$S(P) = i+j+inv(S(P)) = 2+2+5 = 9$ (odd)

$S(Q) = i+j+inv(S(P)) = 2+3+7 = 12$ (even)

NO, because from part (a), in order for Q to be a position reachable from P from a finite number of legal moves, the parity of $S(P)$ and $S(Q)$ must be equal. But the parity of $S(P)$ is odd and the parity of $S(Q)$ is even, so Q is not reachable from position P by a sequence of legal moves.

7

5. (10 points)
Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

Pf: let $H \leq G$. Let $|G| = n$, $|H| = m$, and $H = \{h_1, h_2, \ldots, h_m\}$. Let H have k cosets. Let $g_1, g_2, \ldots, g_k \in G$. The cosets of H are:

$g_1 H = \{g_1 h_1, g_1 h_2, \ldots, g_1 h_m\}$

$g_2 H = \{g_2 h_1, g_2 h_2, \ldots, g_2 h_m\}$, . . . (see front page) →

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x, y) + iv(x, y)$ where $u(x, y), v(x, y)$ satisfy the above system of two equations?

Cauchy-Riemann equations
u (x,y) + iv (x,y) was Riemann's doctoral thesis on theory of complex functions. This led to the conception of a Riemann surface, which introduced topological considerations into analysis

10

7. (10 points) Who discovered the quaternions? What city did that person live in?

William Rowan Hamilton,

Lived in Doblin

10

8. (10 points) What is Heron's formula, what century did Heron live in?

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
1st century AD

10

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

Newton's teacher was John Wallis
Wallis extended algebra into a veritable analysis
Newton studied at cambridge University

6

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?

Leibnitz spent his life in France

0

11. (10 points total)
(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$$\frac{2}{\pi} = \cos\left(\frac{\pi}{4}\right) \cos\left(\frac{\pi}{8}\right) \cos\left(\frac{\pi}{16}\right) \cos\left(\frac{\pi}{32}\right) \cdots$$

10

(b) (5 points) State the names of two people who initiated the use of logarithms

John Neper
Henry Briggs

NAME: (print!) Jennifer Mead

E-Mail address: jennifer.mead@rutgers.edu

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

---

1. 10 (out of 10)
2. 10 (out of 10)
3. 4 (out of 10)
4. 10 (out of 10)
5. 10 (out of 10)
6. 0 (out of 10)
7. 10 (out of 10)
8. 10 (out of 10)
9. 5 (out of 10)
10. 10 (out of 10)
11. 10 (out of 10)

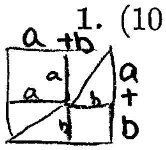
---

total: (out of 110)

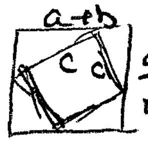
$89 \times 1.134 = \underline{101}$

1. (10 pts.) Give two proofs of the Pythagorean theorem.

a)



$(a+b)^2 = a^2+b^2+\frac{4ab}{2}$



$(a+b)^2 = c^2+\frac{4ab}{2}$

$a^2+b^2+\cancel{\frac{4ab}{2}} = c^2+\cancel{\frac{4ab}{2}}$

$a^2+b^2=c^2$

b)

Area of I + Area of II = Area of III

$Ka^2 + Kb^2 = Kc^2$

$a^2+b^2=c^2$

10

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

Suppose $\sqrt[7]{3}$ is rational. Then $\sqrt[7]{3} = \frac{m}{n}$, $\gcd(m,n)=1$

$3 = \frac{m^7}{n^7} \Rightarrow 3n^7 = m^7$

Then $3 \mid m^7 \Rightarrow 3 \mid m\cdot m\cdot m\cdot m\cdot m\cdot m\cdot m$ so $3 \mid m$

Thus m can be written as $m = 3p$

$\therefore 3n^7 = (3p)^7$

$n^7 = 3^6 p^7 \Rightarrow 3^6 \mid n^7 \Rightarrow 3^6 \mid n\cdot n\cdot n\cdot n\cdot n\cdot n\cdot n \Rightarrow 3^6 \mid n \Rightarrow 3 \mid n$

Thus n can be written as $n = 3q$

But then m & n have a common factor of 3, but we assumed $\gcd(m,n)=1$

Thus $\sqrt[7]{3}$ is irrational.

10

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

The Mandelbrot set is the set of c's for which $z \to z + c$ does not diverge.

$z \to z^2 + c$

2

(b) (5 points) Define the Feigenbaum constant. Explain everything!

$\lim_{n\to\infty} \frac{r_n - r_{n-1}}{r_{n+1} - r_n} \approx 4.669$

2

define r_n!

4. (10 pts. total)

(a) (6 pts.) For any position P in the $(n^2\text{-}1)$-puzzle, let $[i, j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1, 2, 3, \ldots, n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + \; inv\,(\pi(P)) \quad ,$$

where $inv\,(\pi)$ is the number of inversions of the permutation π.
Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd.**
Note: You may use the lemma that if you exchange *any* two elements in a permutation, the number of inversions *always* changes by an odd integer.

Given $S(P)$, $S(Q)$ takes m moves to reach. For each move, $inv(P) - inv(P_{ij})$ is odd and $i+j$ changes by ± 1. Thus, for each move in total $(inv(P) - inv(P_{ij}) + i + j)$ is even. Thus for each move an even number will be "added" to $S(P)$ until we reach $S(Q)$, thereby preserving the parity.

6

(b) (4 pts)
Let

1 2 3 5 9 4 6 7 8 $\qquad\qquad$ 4 2 1 3 5 9 6 7 8

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & \\ 6 & 7 & 8 \end{pmatrix} \quad .$$

Can you reach position ~~P~~ Q from position P, by a sequence of legal moves? Explain!

$inv(P) = 1 + 4$ $\qquad$ $inv(Q) = 3 + 1 + 3$

$S(P) = 5 + 2 + 2 = 9$ $\qquad$ $S(Q) = 7 + 2 + 3 = 12$

No, the parities of $S(P)$ and $S(Q)$ are not the same.

4

5. (10 points)

10

Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

Let $H = \{h_1, h_2, h_3 \ldots h_m\}$ $|H| = m$, $G = \{g_1, g_2, \ldots g_n\}$ $|G| = n$ $\quad H \subseteq G$.
we can find $a_1 \notin H$ and create coset $H_1 = \{a_1 h_1, a_1 h_2, \ldots a_1 h_m\}$
All elements of H_1 are distinct (if $a_1 h_i = a_1 h_j \Rightarrow (a_1^{-1} a_1) h_i = (a_1^{-1} a_1) h_j \Rightarrow h_i = h_j$ contradiction)
H_1 does not intersect H (if $a_1 h_i = h_j \Rightarrow a_1 h_i h_i^{-1} = h_j h_i^{-1} \Rightarrow a_1 = h_i h_j^{-1} \Rightarrow a_1 \in H$, contradiction)
we can then find $a_2 \notin H, H_1$ and form coset H_2 similarly. We can form ~~n~~ of these cosets.
Thus, m

where is this??

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x,y)+iv(x,y)$ where $u(x,y), v(x,y)$ satisfy the above system of two equations?

X

0

7. (10 points) Who discovered the quaternions? What city did that person live in?

William Rowan Hamilton

Dublin?, Ireland

10

8. (10 points) What is Heron's formula, what century did Heron live in?

1st century AD

Area of a triangle: $A = \sqrt{s(s-a)(s-b)(s-c)}$

10

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

Isaac Barrow

Studied at Cambridge University

X

5

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

Born in Leipzig, spent life near Hanover, under King George I

10

11. (10 points total)

(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$$\frac{2}{\pi} = \cos\frac{\pi}{4} \cos\frac{\pi}{8} \cos\frac{\pi}{16} \cos\frac{\pi}{32} \ldots$$

~~5~~

10

(b) (5 points) State the names of two people who initiated the use of logarithms

John Napier + Henry Briggs.

/

NAME: (print!) REUBEN RIOS

E-Mail address: RS2111@SCARLETMAIL.RUTGERS.EDU

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

---

1. 5 (out of 10)
2. 5 (out of 10)
3. 10 (out of 10)
4. 8 (out of 10)
5. 4 (out of 10)
6. 6 (out of 10)
7. 10 (out of 10)
8. 10 (out of 10)
9. 6 (out of 10)
10. 10 (out of 10)
11. 10 (out of 10)

---

total: 84 (out of 110) $\cdot 1.134 = 95.5$

5. $G = \left\{ \begin{pmatrix} 1&2&3 \\ 1&2&3 \end{pmatrix}, \begin{pmatrix} 1&2&3 \\ 1&3&2 \end{pmatrix}, \begin{pmatrix} 1&2&3 \\ 2&1&3 \end{pmatrix}, \begin{pmatrix} 1&2&3 \\ 2&3&1 \end{pmatrix}, \begin{pmatrix} 1&2&3 \\ 3&1&2 \end{pmatrix}, \begin{pmatrix} 1&2&3 \\ 3&2&1 \end{pmatrix} \right\}$

$H = \left\{ \begin{pmatrix} 1&2&3 \\ 1&2&3 \end{pmatrix}, \begin{pmatrix} 1&2&3 \\ 2&3&1 \end{pmatrix}, \begin{pmatrix} 1&2&3 \\ 3&1&2 \end{pmatrix} \right\}$

COSET $\begin{pmatrix} 1&2&3 \\ 1&3&2 \end{pmatrix} H = \left\{ \begin{pmatrix} 1&2&3 \\ 1&3&2 \end{pmatrix}, \begin{pmatrix} 1&2&3 \\ 2&1&3 \end{pmatrix}, \begin{pmatrix} 1&2&3 \\ 3&2&1 \end{pmatrix} \right\}$

$i \neq j$

$gh_i = gh_j$

$(g^{-1}gh_i) = (g^{-1}gh_j)$

$eh_i = eh_j$

$h_i = h_j$

$gh_i = h_j$

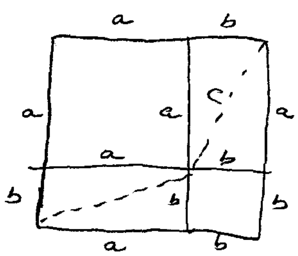
$g = h_j h_i^{-1}$

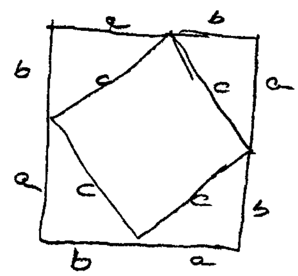
$h_i = gh_j$

$h_j^{-1}h_i = g$

$H \cap gH = \emptyset$

1. (10 pts.) Give two proofs of the Pythagorean theorem.





$a^2+b^2=4\left(\frac{1}{2}ab\right)=(a+b)^2$

$a^2+2ab+b^2=a^2+b^2+2ab$

$c^2+4\left(\frac{1}{2}ab\right)=(a+b)^2$

$a^2+b^2=c^2$

5

2nd proof ?

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

LEMMA: EVERY INTEGER n CAN BE WRITTEN AS $3^i a$ WHERE $\gcd(a,3)=1$ FOR SOME INTEGER $i \geq 0$

$\sqrt[7]{3}=\frac{m}{n}$

$3=\frac{m^7}{n^7}$

$m^7=3n^7$

$m=3^i a$

$n=3^j b$

$(3^i a)^7=3(3^j b)^7$

$3^{7i}a^7=3^{7j+1}b^7$

POWER OF 3 ON THE LEFT SIDE IS ODD AND THE ONE ON THE LEFT IS EVEN, SINCE THEY ARE SUPPOSE TO BE EQUAL, THERE'S A CONTRADICTION.

5

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

MANDELBROT SET IS A SET THAT EXHIBITS A REPEATING PATTERN DISPLAYED AT EVERY SCALE $S_c(z)=z^2+c$ DOES NOT DIVERGE WHEN INTEGRATED FROM $z=0$.

5

(b) (5 points) Define the Feigenbaum constant. Explain everything!

FEIGENBAUM CONSTANT IS THE LIMITING RATIO OF EACH BIFURCATION INTEGRAL TO THE NEXT BETWEEN EVERY PERIOD DOUBLING OF A ONE-PARAMETER MAP $x_{i+1}=f(x_i)$.

$\delta=\lim_{n\to\infty}\frac{r_n-r_{n-1}}{r_{n+1}-r_n}=4.669201609\ldots$

5

4. (10 pts. total)
(a) (6 pts.) For any position P in the (n^2-1)-puzzle, let $[i, j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1, 2, 3, \ldots, n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + \ inv\,(\pi(P)) \quad ,$$

where $inv\,(\pi)$ is the number of inversions of the permutation π.
Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd.**
Note: You may use the lemma that if you exchange *any* **two** elements in a permutation, the number of inversions *always* changes by an odd integer.

LEMMA: IF YOU EXCHANGE ANY TWO ELEMENTS IN A PERMUTATION THE NUMBER OF INVERSIONS ALWAYS CHANGES BY AN ODD INTEGER.

SLIDING HORIZONTAL PARITY OF PERMUTATION AND TAXI CAB DISTANCE CHANGES SLIDING VERTICAL n^2 CHANGED WITH ANOTHER, NUMBER OF INVERSIONS CHANGED, PARITY OF NUMBER OF INVERSIONS + ROW+COLUMN NUMBER REMAINS SAME. THUS ITS IMPOSSIBLE TO GET IT,

6

(b) (4 pts)
Let

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & \\ 6 & 7 & 8 \end{pmatrix} \quad .$$

Can you reach position ~~P~~ Q from position P, by a sequence of legal moves? Explain!

$S(P) = INV(\pi) + i + j$	$S(Q) = INV(\pi) + i + j$
$i = 2, j = 2$	$i = 2, j = 3$
$\pi = 1\,2\,3\,5\,9\,4\,6\,7\,8$	$\pi = 4\,2\,1\,3\,5\,9\,6\,7\,8$
$INV(\pi) = 5$	$INV(\pi) = 7$
$S(P) = 5 + 2 + 2 = 9$	$S(Q) = 7 + 2 + 3 = 12$

what is the answer? Yes or No?

2

5. (10 points)
Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

You did an example!

4

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x, y) + iv(x, y)$ where $u(x, y), v(x, y)$ satisfy the above system of two equations?

CAUCHY RIEMANN EQUATION

IT'S EULER'S EQUATION: COSΘ+iSINΘ

6

7. (10 points) Who discovered the quaternions? What city did that person live in?

WILLIAM ROWAN HAMILTON

DUBLIN

10

8. (10 points) What is Heron's formula, what century did Heron live in?

AREA OF TRIANGLE $= \sqrt{s(s-a)(s-b)(s-c)}$

1ST CENTURY

10

9. (10 points) Where did Isaac Newton study? Who was his teacher? ~~What unusual action did that teacher do~~? What was Newton's position after he left Cambridge?

CAMBRIDGE

ISAAC BARROW

6

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

LEIPIG

COURT OF HANOVER

KING GEORGE I

10

11. (10 points total)

(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$\frac{2}{\pi} = \cos(\frac{\pi}{4})\cos(\frac{\pi}{8})\cos(\frac{\pi}{16})\cos(\frac{\pi}{32})\ldots$

10

(b) (5 points) State the names of two people who initiated the use of logarithms

JOHN NEPER

HENRY BRIGGS

NAME: (print!) Lauren Squillace

E-Mail address: lns65@scarletmail.rutgers.edu

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

1. 10 (out of 10)
2. 10 (out of 10)
3. 10 (out of 10)
4. 9 (out of 10)
5. 10 (out of 10)
6. 0 (out of 10)
7. 0 (out of 10)
8. 10 (out of 10)
9. 8 (out of 10)
10. 10 (out of 10)
11. 6 (out of 10)

total: (out of 110)

$83 \cdot 1.134 =$

94.5

5.) Suppose that G is a group and that H is a subgroup of G. We define the following cosets:

$g_1H = \{g_1h_1, g_1h_2, \ldots, g_1h_m\}$

$g_2H = \{g_2h_1, g_2h_2, \ldots, g_2h_m\}$

$\vdots$

$g_nH = \{g_nh_1, g_nh_2, \ldots, g_nh_m\}$

Lemma 1: $H \cap g_iH = \emptyset$.

Suppose that $h_k = g_ih_j$

$h_kh_j^{-1} = g_ih_jh_j^{-1} = g_i \notin H$

this is a contradiction because $h_k h_j^{-1} \in H$, but $g_i \notin H$.

Lemma 2: Each g_ih_j is distinct.

Suppose that $g_ih_j = g_ih_k$

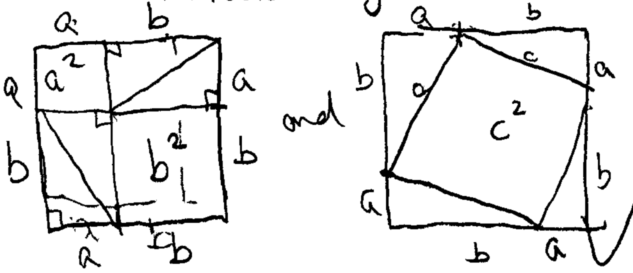
$g_i^{-1}g_ih_j = g_i^{-1}g_ih_k$

$h_j = h_k$, which is a contradiction.

By these lemmas, there are $m \times n$ elements in G and m elements in H. So we have $|G| = m \times n = |H| \times n$, which implies that $\frac{|G|}{|H|} = n$, which is an integer.

1. (10 pts.) Give two proofs of the Pythagorean theorem.

1.) We will consider two squares with side length $a+b$.

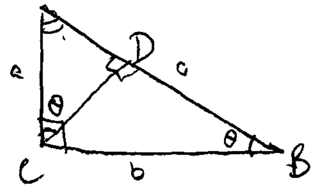


The right triangles within both squares have leg lengths a, b and hypotenuse of length c. The areas of both squares must be equal, so we have

$$a^2+b^2+4\left(\tfrac{1}{2}ab\right)=c^2+4\left(\tfrac{1}{2}ab\right),$$

which implies that $a^2+b^2=c^2$.

2.)



$\triangle ABC \sim \triangle ACD \sim \triangle CBD$

There is a theorem that says that there exists a constant k such that if similar triangles have longest side A, the triangle's area is KA^2.

Using this theorem, we have

$ka^2+kb^2=kc^2$; which implies that $a^2+b^2=c^2$.

(Since a is the longest side of $\triangle ADC$, b is the longest side of $\triangle DBC$, etc. And the area of the larger triangle is the sum of the two smaller triangles.)

10

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

Suppose that $\sqrt[7]{3}$ is rational. Then $\sqrt[7]{3}=\frac{m}{n}$, for some integers m, n. So $3=\frac{m^7}{n^7}$ and $3n^7=m^7$. Since every integer can be written as a product of primes, $m=3^i\bar{m}$, where i is a nonnegative integer and $\bar{m}$ is not divisible by 3. Also $n=3^j\bar{n}$, where j is a nonnegative integer and $\bar{n}$ is not divisible by 3.

Then we have
$$3(3^j\bar{n})^7=(3^i\bar{m})^7$$
$$3^{7j+1}\bar{n}^7=3^{7i}\bar{m}^7$$

Since $\bar{m}$ is not divisible by 3, neither is $\bar{m}^7$. Similar for $\bar{n}$. The number of factors of 3 on the left when taken mod 7, is 1. The number of factors of 3 on the right is 0 when taken mod 7. Thus, both sides cannot be equal.

10

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

The Mandelbrot set is the set of all c such that $z\to z^2+c$ converges to one fixed point or a finite orbit.

5

(b) (5 points) Define the Feigenbaum constant. Explain everything!

The Feigenbaum constant is $\lim_{n\to\infty}\frac{r_n-r_{n-1}}{r_{n+1}-r_n}\approx 4.669$, where r_i is the ith point at which the period of $x\to rx(1-x)$ doubles.

5

4. (10 pts. total)

(a) (6 pts.) For any position P in the (n^2-1)-puzzle, let $[i,j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1,2,3,\ldots,n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + \ inv\,(\pi(P)) \quad ,$$

where $inv\,(\pi)$ is the number of inversions of the permutation π.

Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd.**

Note: You may use the lemma that if you exchange *any* **two** elements in a permutation, the number of inversions *always* changes by an odd integer.

Suppose that Q is reachable from P. Then there are a finite number of legal moves that can transform P into Q. In any legal move, the blank is swapped with an adjacent tile, so $(i+j)$ changes by ± 1.

Lemma: If you exchange any two elements in a permutation, π to form new permutation π_{ij}, then $inv(\pi_{ij}) - inv(\pi)$ is odd.

Consider permutation π corresponding to P. A legal move swaps two elements of the permutation to form a new permutation π_{ij}. Let P' be the board after one legal move. Then $S(P') = S(P) \pm 1 + a$, where a is odd, by our lemma. Since a is odd, $\pm 1 + a$ is even. So $S(P)$ and $S(P')$ have the same parity. This proof can be repeated for the number of legal moves it takes to get from P to Q, so the parity of $S(Q)$ is equal to the parity of S.

(b) (4 pts)

Let

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & \square \\ 6 & 7 & 8 \end{pmatrix}$$

$i=2, j=3$

Can you reach position ~~P~~ Q from position P, by a sequence of legal moves? Explain!

$S(P) = inv(123594678) + 2 + 2$
$= 5 + 4 = 9$

$inv(Q) = inv(421359678) + 3 + 2$
$= 7 + 5 = 12$

No
~~Yes~~, you can't Reach Q from P because the parity of S for each board ~~is~~ are not the same; ~~they are both odd.~~

right Way!

5. (10 points)

Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

Su ppose

6

3

10

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad ; \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x,y) + iv(x,y)$ where $u(x,y), v(x,y)$ satisfy the above system of two equations?

Hamiltonian System

X 0

7. (10 points) Who discovered the quaternions? What city did that person live in?

Lagrange, Turin

X 0

8. (10 points) What is Heron's formula, what century did Heron live in?

$A = \sqrt{s(s-a)(s-b)(s-c)}$

1st century AD

✓ 10

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

Cambridge University. ✓

His teacher was Isaac Barrow. He said that Newton was his superior and gave him his professorship. ✓ ; Warden of the mint

✓ 8

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?. — George I ✓

He was born in Leipzig. Spent most of his life in Hanover. ✓

10

11. (10 points total)

(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$$\frac{2}{\pi} = \frac{1 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7 \cdots}{2 \cdot 2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8 \cdot 8 \cdots}$$

X

this is Wallis'

(b) (5 points) State the names of two people who initiated the use of logarithms

John Napier ✓ , Briggs ✓

6

/

NAME: (print!) Tyler Volpe

E-Mail address: Tyler.volpe@rutgers.edu

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

---

1. 10 (out of 10)
2. 10 (out of 10)
3. 8 (out of 10)
4. 10 (out of 10)
5. 10 (out of 10)
6. 7 (out of 10)
7. 0 (out of 10)
8. 10 (out of 10)
9. 3 (out of 10)
10. 3 (out of 10)
11. 10 (out of 10)

---

total: 81 (out of 110) $\cdot 1.134 =$

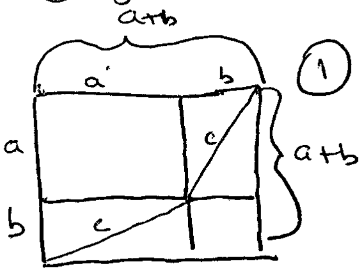
91.85

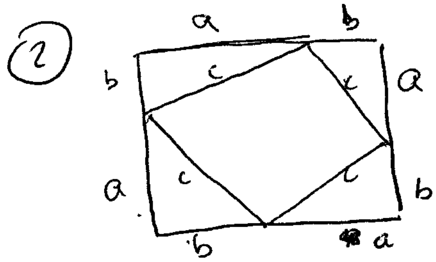
(5) Let H be a subgroup of a finite group G. Then if $H = G$, $\frac{|G|}{|H|} = 1$, and we are done. Otherwise, pick $g_1 \in G/H$. Then notice that $g_1 h_i \neq g_1 h_j$ for $h_i \neq h_j$. To see this, notice $g_1 h_i = g_1 h_j \Rightarrow g_1^{-1} g_1 h_i = g_1^{-1} g_1 h_j \Rightarrow h_i = h_j$. Also notice $g_1 h_i \neq h_j$ for any $h_i \neq h_j$. This is true because $g_1 h_i = h_j \Rightarrow g_1 = h_j h_i^{-1}$. But H is a group, so $g_1 = h_j h_i^{-1} \in H$, which is not possible. It follows that $|g_1 H| = |H|$, and since they do not share elements, $|H \cup g_1 H| = 2|H|$. If $H \cup g_1 H = G$, then again $\frac{|G|}{|H|} = 2$, and we are done. If not, continue the process, taking $g_2 \in G/(H \cup g_1 H)$. Repeat this until there are no remaining elements of G, and notice that then $G = H \cup g_1 H \cup g_2 H \cup \dots \cup g_n H$, with each $g_i H$ not overlapping with $g_j H$. So $|G| = n|H|$. That is, $\frac{|G|}{|H|}$ is an integer. $\square$

$\checkmark$

1. (10 pts.) Give two proofs of the Pythagorean theorem.

Construct two squares of side length $a+b$ as follows:





Then by ①, $(a+b)^2 = a^2+b^2+4(\text{area of }\Delta)$.

By ②, $(a+b)^2 = c^2+4(\text{area of }\Delta)$.

It follows that $a^2+b^2=c^2$.

10

Every family of similar triangles has area proportional to the square of the longest side. By image 3, $ka^2+kb^2=kc^2$ (since triangles I, II, III are similar). Then $a^2+b^2=c^2$. □

③

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

Suppose not. Then we may write $3^{1/7}=\frac{m}{n}$ where $\gcd(m,n)=1$. But then $3n^7=m^7$. Hence $3|m^7$, so $3|m$. (by Euclid's lemma, since 3 is prime). Then $m=3k$ for some $k\in\mathbb{N}$. So $3n^7=(3k)^7 \Rightarrow n^7=3^6k^7$. So $3|n^7$, and as above, $3|n$. This contradicts the fact that $\gcd(m,n)=1$. So $\sqrt[7]{3}$ must be irrational. □

10

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

$$M=\{c\in\mathbb{C} \mid \text{the sequence defined by } z_0=0,\ z_n=z_{n-1}^2+c \text{ is bounded}\}.$$

5

(b) (5 points) Define the Feigenbaum constant. Explain everything!

③ $\lambda=\dfrac{r_n-r_{n-1}}{r_{n+1}-r_n}\approx 4.669\ldots$ Where r_n is the real number at which the ~~sequence $z_0=0$, $z_{n-1}^2+c=z_n$~~ changes from having 2^{n-1} limiting values to 2^n limiting values. (i.e., $r<r_1 \Rightarrow 2^0=1$ limit, $r>r_1 \Rightarrow 2^1=2$ limits).

$x_{n+1}=rx_n(1-x_n)$

4. (10 pts. total)
(a) (6 pts.) For any position P in the (n^2-1)-puzzle, let $[i,j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1,2,3,\dots,n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + \; inv\,(\pi(P)) \quad ,$$

where $inv\,(\pi)$ is the number of inversions of the permutation π.
Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd**.
Note: You may use the lemma that if you exchange *any* **two** elements in a permutation, the number of inversions *always* changes by an odd integer.

Notice that any legal move does not change the parity of $S(P)$.
In a row-move, $j'=j\pm 1$, $i'=i$, $inv(\pi(P')) = inv(\pi(P)) \pm$ odd number. by the lemma
So $S(P') = inv(\pi(P'))+i'+j' = inv(\pi(P'))+i+j \pm 1 \pm$ odd number $=$ $S(P) \pm$ even number, so parity is preserved. Similarly for a column move (here $i'=i\pm1$, $j'=j$). Since this is true for each move, then after any number of moves we still have parity of $S(P)$ = parity of $S(Q)$.

(b) (4 pts)
Let

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & \\ 6 & 7 & 8 \end{pmatrix} \quad .$$

10

Can you reach position ~~P~~ Q from position P, by a sequence of legal moves? Explain!

$inv(\pi(P)) = inv(1\,2\,3\,5\,9\,4\,6\,7\,8) = 5$
(1) (4)

$inv(\pi(Q)) = inv(4\,2\,1\,3\,5\,9\,6\,7\,8) = 7$.
(3) (1) (3)

$S(P) = 2+2+5 = 9$ (odd).
$S(Q) = 2+3+7 = 12$ (even).
Since parity of $S(Q) \neq$ parity of $S(P)$, this is not reachable by any # of legal moves.

5. (10 points)
Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

10

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x,y) + iv(x,y)$ where $u(x,y), v(x,y)$ satisfy the above system of two equations?

" Cauchy - Riemann equations
Riemann wrote his docteral thesis on these functions

7

7. (10 points) Who discovered the quaternions? What city did that person live in?

Galois, Paris.

0

8. (10 points) What is Heron's formula, what century did Heron live in?

area of a triangle $A = \sqrt{s(s-a)(s-b)(s-c)}$; 1^{st}

10

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

Cambridge ; Pierre de Mauper tuis ; proved existence of God

3

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

Leipzig, [illegible] Paris ;

3

11. (10 points total)
(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$$\frac{2}{\pi} = \cos\frac{\pi}{4} \cos\frac{\pi}{8} \cos\frac{\pi}{16} \cos\frac{\pi}{32} \cdots$$

5

(b) (5 points) State the names of two people who initiated the use of logarithms

Napier, Briggs

5

/

NAME: (print!) Kevin Lin

E-Mail address: its.kevinlin @ rutgers.edu

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

$G = \{g_1 g_2 \cdots g_m\}$
$H = \{h_1 h_2 \cdots h_n\}$

Do not write below this line (office use only)

---

1. 10 (out of 10)
2. 9 (out of 10)
3. 7 (out of 10)
4. 10 (out of 10)
5. 10 (out of 10)
6. 0 (out of 10)
7. 10 (out of 10)
8. 7 (out of 10)
9. 0 (out of 10)
10. 4 (out of 10)
11. 10 (out of 10)

---

total: 77 (out of 110) $\cdot 1.134$

≈ 87.5

(5) Let $|G| = m$; H subgroup of G, $|H| = n$.

Let $g_1 \in G$, Then the coset ...

$g_1 H = \{g_1 h_1, g_1 h_2 \cdots g_1 h_n\}$.

All the $g_1 h_i$'s are unique.

Pf. Assume not, that $g_1 h_i = g_1 h_j$ $i \neq j$.
Then $g_1^{-1} g_1 h_i = g_1^{-1} g_1 h_j$
$e h_i = e h_j$
$h_i = h_j$ a contradiction

Also they do not overlap

Pf. Assume not. that $g_1 h_i = h_j$
Then $g_1 h_i h_i^{-1} = h_j h_i^{-1}$
$g_1 e = h_j h_i^{-1}$
$g_1 = h_j h_i^{-1}$

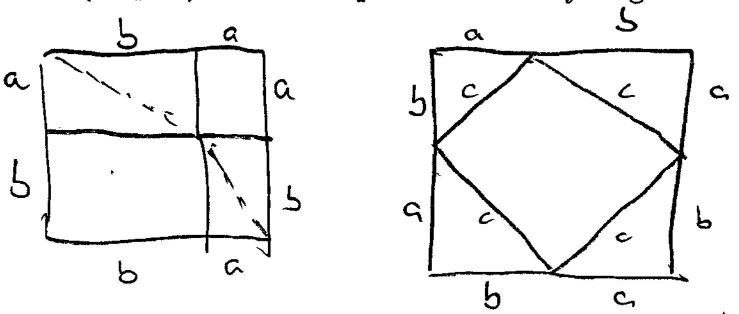
Since H is a group and closed then $g_1 = h_j h_i^{-1} \in H$ which is a contradiction

Thus, since there is no overlapping, we can count the # of elements

$$n \left\{ \begin{matrix} g_1 H \\ g_2 H \\ \vdots \\ g_m H \end{matrix} \right.$$

So $\frac{|G|}{|H|} = \frac{m}{n} \in \mathbb{Z}$

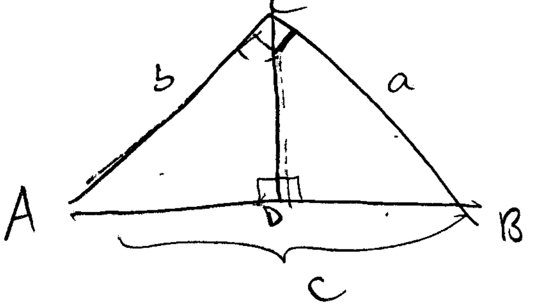
1. (10 pts.) Give two proofs of the Pythagorean theorem.



Since the four triangles are equal, it must be the case that the big triangle on the right is equal to the sum of the two triangles on the left

$$a^2+b^2+4(\tfrac{1}{2}ab)=c^2+4(\tfrac{1}{2}ab)$$

$$a^2+b^2=c^2$$



Something with similar triangles

Lemma producing a k^2 term for each Δ.
similar triangles

Conclusion: $\Delta ACD \sim \Delta BCD \sim \Delta ABC$.
Cancel out the k's and get $a^2+b^2=c^2$

10

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

Assume not, that it is rational. Then $\sqrt[7]{3}=\frac{a}{b}$, $b\neq 0$ $\gcd(a,b)=1$.
Then $3=\frac{a^7}{b^7}$ or $3b^7=a^7$. Now by a lemma extended from the Fundamental Thm of Arithmetic $b=3^i n$ and $a=3^j m$
with $i\geq 0$, $j\geq 0$, $\gcd(3,n)=1$, $\gcd(3,m)=1$. Then $3(3^i n)^7=(3^j m)^7$
or $3^{7i+1}n^7=3^{7j}m^7$. Note that this is impossible due to the fact that 3^{7i+1} can never equal 3^{7j}.

why?

9

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

Define $z_{n+1}=z_n^2+c$, $z_0=0$,

An element c is in the mandelbrot set if the sequence of $z_0, z_1, z_2, z_3 \ldots$ does not diverge

5

(b) (5 points) Define the Feigenbaum constant. Explain everything!

2

≈ 4.669

limit of population growth and decay (e.g. rabbits & foxes) and what that limit tends to

4. (10 pts. total)
(a) (6 pts.) For any position P in the $(n^2$-1)-puzzle, let $[i, j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1, 2, 3, \ldots, n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + \ inv\,(\pi(P)) \quad ,$$

where $inv\,(\pi)$ is the number of inversions of the permutation π.
Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd**.
Note: You may use the lemma that if you exchange *any* **two** elements in a permutation, the number of inversions *always* changes by an odd integer.

Let P be a position. Then let P' be the position after moving the blank space one tile. By lemma an exchange of any two elements in a permutation changes the inversions by an odd integer. Now if we move vertically, $i \pm 1$ and j stays the same, If we move horizontally, $j \pm 1$ and i stays the same. Thus, odd ± 1 = even.
So if $S(P)$ is even, then $S(P')$ = even and if $S(P)$ odd, $S(P')$ odd Since parity preserved. Now, after a finite amount of moves from position P' we can reach position Q. Using the same steps, the parity of $S(P)$ is preserved.

6

(b) (4 pts)
Let

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & \\ 6 & 7 & 8 \end{pmatrix} \quad .$$

Can you reach position ~~R~~ Q from position P, by a sequence of legal moves? Explain!

(0)(0)(0)(1)(4)(0)(0)(0)(0)
$\pi_P = 1\ 2\ 3\ 5\ 9\ 4\ 6\ 7\ 8$
$inv(\pi_P) = 5$
$S(P) = 5 + 2+2 = 9$

$\pi_Q = 4\ 2\ 1\ 3\ 5\ 9\ 6\ 7\ 8$
(3)(1)(0)(0)(0) (3)(0)(0)(0)
$inv(\pi_Q) = 7$
$S(Q) = 7 + 2+3 = 12$

4

Since they are different parities, impossible to reach Q

5. (10 points)
Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

10

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

Hamilton-Jacobi equation X

0

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x,y) + iv(x,y)$ where $u(x,y), v(x,y)$ satisfy the above system of two equations?

"Characteristic / principal" functions

0

Can't find in notes.

7. (10 points) Who discovered the quaternions? What city did that person live in?

William Rowan Hamilton ✓ Dublin ✓

10

8. (10 points) What is Heron's formula, what century did Heron live in?

Area triangle: $A = \sqrt{s(s-a)(s-b)(s-c)}$ ✓

7

around Alexandria time so circa 4th century BC X

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

He studied "fluxions" Aka what he referred to as calculus

0

X

teacher / professor ?

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

born Leipzig ✓ spent most of life (besides math) promoting the unity of Germany X

4

11. (10 points total)

(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$$\frac{2}{\pi} = \cos\frac{\pi}{4}\cos\frac{\pi}{8}\cos\frac{\pi}{16}\cos\frac{\pi}{32}\cdots\cdots$$

✓ 5

(b) (5 points) State the names of two people who initiated the use of logarithms

John Neper, Henry Briggs ✓

5

NAME: (print!) Joshua Pomerantz

E-Mail address: jmp510 @scarletmail.rutgers.edu

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

---

1. 10 (out of 10)

2. 10 (out of 10)

3. 7 (out of 10)

4. 10 (out of 10)

5. 10 (out of 10)

6. 0 (out of 10)

7. 0 (out of 10)

8. 0 (out of 10)

9. 10 (out of 10)

10. 10 (out of 10)

11. 10 (out of 10)

---

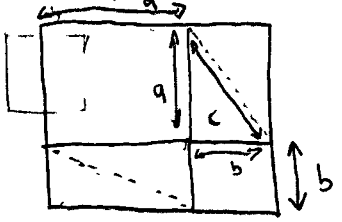
total: (out of 110)

77 · 1.134 = 87.5

1. (10 pts.) Give two proofs of the Pythagorean theorem.

Geometric proof #1

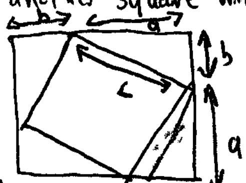
Draw a square that's $(a+b) \times (a+b)$



Call the diagonals of the $a \times b$ rectangles c. The total area of this $(a+b) \times (a+b)$ square is

$(a+b)^2 = a^2 + b^2 + 4 \cdot$ Area of triangles

Draw another square with $a+b$ side length



the length of the sides of the inscribed square is c

The area of the second square is

$(a+b)^2 = c^2 + 4 \cdot$ Area of triangles

Thus

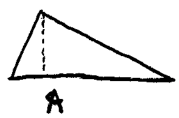
$a^2+b^2+4 \cdot$ Area of triangles $= c^2 + 4 \cdot$ Area of triangles

Thus $a^2+b^2=c^2$

10

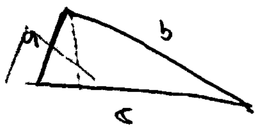
Geometric Proof #2

Lemma: For a family of similar triangles, there is a constant K such that the area of each triangle is K times the length of the triangles longest side, i.e.



Area $= K A^2$

Take 3 similar triangles whose longest sides are a, b and construct a third whose longest side is c



Then the area of the figure is

$Ka^2 + Kb^2 = Kc^2$

Thus $a^2+b^2=c^2$

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

Assume $\sqrt[7]{3} = \frac{m}{n}$ for some $m, n \in \mathbb{N}$. Then $3n^7 = m^7$.

Since all natural numbers can be written as multiples of primes, we can write

$m = 3^i \bar{m}$, $n = 3^j \bar{n}$ where $\bar{n}, \bar{m}$ are not multiples of 3.

Thus

$m^7 = (3^i \bar{m})^7 = 3^{7i} \bar{m}^7$ and $n^7 = (3^j \bar{n})^7 = 3^{7j} \bar{n}^7$

Thus by substitution

$3 \cdot 3^{7j} \bar{n} = 3^{7i} \bar{m}$. This implies that $3^{7j+1} = 3^{7i}$, implying further that $7j+1 = 7i$. But this is impossible! Thus $\sqrt[7]{3}$ is not rational.

10

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

The set of all c such that the sequence $z_0 = 0$; $z_{k+1} = z_k^2 + c$ does not go to infinity.

5

(b) (5 points) Define the Feigenbaum constant. Explain everything!

$$\lim_{n \to \infty} \frac{r_n - r_{n-1}}{r_{n+1} - r_n} = 4.6692$$

what is r_n?

2

~~4.~~ (10 pts. total)
~~(a)~~ (6 pts.) For any position P in the $(n^2$-1)-puzzle, let $[i, j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1, 2, 3, \ldots, n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + \, inv\,(\pi(P)) \quad ,$$

where $inv\,(\pi)$ is the number of inversions of the permutation π.
Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd**.
Note: You may use the lemma that if you exchange *any* two elements in a permutation, the number of inversions *always* changes by an odd integer.

Starting from P, all of our legal moves (of which there are at least 2 and at most 4) involve exchanging two elements of the permutation π associated with P. Thus if we can reach P^* from P in one legal move, calling π^* the permutation associated with P^*, then $inv(\pi^*) = inv(\pi) + K$ where K is an odd number. This follows from the lemma.
In each legal move we change the row or column index by 1. Thus $i^* + j^* = i + j \pm 1$
Therefore $S(P^*) = inv(\pi^*) + i^* + j^* = inv(\pi) + K + i + j \pm 1 = S(P) + K \pm 1$. Thus for any single legal move, we change S by an even number. Inductively, any finite number of legal moves resulting in a position Q renders $S(Q) = S(P) +$ some even number. Thus S(P) and S(Q) have the same parity.

6

~~(b)~~ (4 pts)
~~Let~~

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & \\ 6 & 7 & 8 \end{pmatrix} \quad .$$

Can you reach position ~~P~~ Q from position P, by a sequence of legal moves? Explain!

$\pi =$ 1 2 3 5 9 4 6 7 8
(0) (0) (0) (1) (4) (0) (0) (0) (0)

$inv(\pi) = 5$

$S(P) = 5 + 2 + 2 = 9$

$\pi^* =$ 4 2 1 3 5 9 6 7 8
(3) (1) (0) (0) (0) (3) (0) (0) (0)

$inv(\pi^*) = 7$

$S(Q) = 7 + 2 + 3 = 12$

By the result of (a), we cannot reach Q from P by a sequence of legal moves, since the parities are not the same.

4

~~5.~~ (10 points)
Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

10

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x,y) + iv(x,y)$ where $u(x,y), v(x,y)$ satisfy the above system of two equations?

0

7. (10 points) Who discovered the quaternions? What city did that person live in?

0

8. (10 points) What is Heron's formula, what century did Heron live in?

0

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

Newton studied at Cambridge under Isaac Barrow who in 1669 yielded the Lucasian professorship to his pupil. After leaving Cambridge, Newton became Warden, and later master, of the Mint.

10

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

Leipzig. He spent most of his life near the court of Hanover in service of the Dukes, one of whom became King George I of England.

10

11. (10 points total)

(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$$\frac{2}{\pi} = \cos\frac{\pi}{4}\cos\frac{\pi}{8}\cos\frac{\pi}{16}\cos\frac{\pi}{32}\cdots$$

10

(b) (5 points) State the names of two people who initiated the use of logarithms

John Neper, Henry Briggs, and further Ezechiel De Decker, who published the first table

/

NAME: (print!) André Silva Dias 24-4-17

E-Mail address: aas292@scarletmail.rutgers.edu

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

---

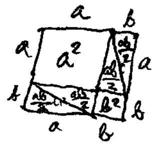
1. 5 (out of 10)
2. 10 (out of 10)
3. 6 (out of 10)
4. 10 (out of 10)
5. 5 (out of 10)
6. 0 (out of 10)
7. 0 (out of 10)
8. 10 (out of 10)
9. 10 (out of 10)
10. 10 (out of 10)
11. 10 (out of 10)

---

total: (out of 110)

76 · 1.134 = 86.5

1. (10 pts.) Give two proofs of the Pythagorean theorem.

By geometry, we have  $a^2+2ab+b^2$. If we rearrange these shapes, we can get another large square of the same area:  c^2+2ab. Since the areas are the same, we can set them equal: $a^2+\cancel{2ab}+b^2=c^2+\cancel{2ab} \to$
$\to a^2+b^2=c^2$.

5

2nd proof?

0

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

Suppose not; suppose $\sqrt[7]{3}=\frac{m}{n}$ for some relatively prime integers m,n. Then $3=\frac{m^7}{n^7}$, and $3n^7=m^7$. Since any number can be written as a product of primes by the theorem discussed in class, let $n=3^i a$ and $m=3^j b$ for $i,j=0,1,2,\dots$ and $i\neq j$. Then $3(3^i a)^7=(3^j b)^7 \to 3(3^{7i}a^7)=3^{7j}b^7 \to 3^{7i+1}a^7=3^{7j}b^7$. $\therefore$ $7i+1=7j$ is a contradiction, so $\sqrt[7]{3}$ must be irrational.

10

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

The Mandelbrot set is a fractal shape on the complex plane that is found by using the mapping $z \to z^2+c$ for a given parameter c.

2

(b) (5 points) Define the Feigenbaum constant. Explain everything!

The Feigenbaum constant is the limiting value of the ratio between the two consecutive ~~integrals~~ numbers in period doubling: $\lim_{n\to\infty}\frac{x_n-x_{n-1}}{x_{n+1}-x_n}=4.6679\ldots$

4

4. (10 pts. total)
(a) (6 pts.) For any position P in the (n^2-1)-puzzle, let $[i, j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1, 2, 3, \ldots, n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + \mathit{inv}\,(\pi(P)) \quad ,$$

where $\mathit{inv}\,(\pi)$ is the number of inversions of the permutation π.
Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd**.
Note: You may use the lemma that if you exchange *any* **two** elements in a permutation, the number of inversions *always* changes by an odd integer.

In the (n^2-1)-puzzle, there are two legal moves: a horizontal slide and a vertical slide. In a horizontal slide, j changes by ± 1 (odd), and the number of inversions changes by an odd number by the lemma; therefore, $S(Q)$ has the same parity as $S(P)$. In a vertical slide, i changes by ± 1 (odd), and the number of inversions changes by an odd number again, by the lemma; therefore, $S(Q)$ again has the same parity as $S(P)$. Thus, either $S(P)$ and $S(Q)$ are both even or both odd.

6

(b) (4 pts)
Let

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & \\ 6 & 7 & 8 \end{pmatrix} \quad .$$

Can you reach position ~~P~~ Q from position P, by a sequence of legal moves? Explain!

$$S(P) = 2 + 2 + (1+4) = 4 + (5) = 9$$
$$S(Q) = 2 + 3 + (3 + 1 + 3) = 5 + (7) = 12$$

Position Q cannot be reached from position P since $S(P)$ does not have the same parity as $S(Q)$.

4

5. (10 points)
Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

Suppose $G = \{g_1, g_2, \ldots, g_m\}$ and $H = \{h_1, h_2, \ldots, h_n\}$. For all elements of H, $h_i \neq h_j$ for $i \neq j$. There is also no overlap between H and its cosets g_1H, g_2H, etc.
Because of this, $\frac{|G|}{|H|} = \frac{m}{n}$ = an integer.

why? You had to prove it

5

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x,y) + iv(x,y)$ where $u(x,y), v(x,y)$ satisfy the above system of two equations?

X 0

Spent more than 15 minutes looking for these and still can't find them

7. (10 points) Who discovered the quaternions? What city did that person live in?

don't even know what that is or if it is a term from geometry, from calculus, etc.

0

8. (10 points) What is Heron's formula, what century did Heron live in?

$A = \sqrt{s(s-a)(s-b)(s-c)}$; 1^{st} century A.D.

10

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

Cambridge; Isaac Barrow; Barrow completely explained by geometry that differentiation and integration are inverses; Newton, after being Lucasian Professor in Cambridge was warden, then master of the mint, and then knighted by Queen Anne

10

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

Leipzig; Hanover; King George I

10

11. (10 points total)
(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$\frac{2}{\pi} = \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{16}\right)\cos\left(\frac{\pi}{32}\right)\ldots$

10

(b) (5 points) State the names of two people who initiated the use of logarithms

Napier and Briggs

NAME: (print!) Suiliang Ma

E-Mail address: sm1659@scarletmail.rutgers.edu

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

---

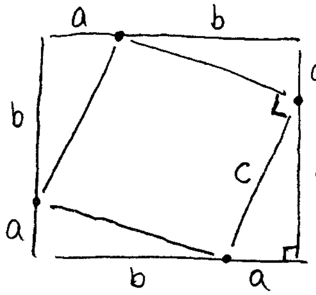
1. 10 (out of 10)
2. 9 (out of 10)
3. 3 (out of 10)
4. 10 (out of 10)
5. 6 (out of 10)
6. 5 (out of 10)
7. 10 (out of 10)
8. 7 (out of 10)
9. 3 (out of 10)
10. 3 (out of 10)
11. 10 (out of 10)

---

total: (out of 110)

76 · 1.134 = 86.5

1. (10 pts.) Give two proofs of the Pythagorean theorem.

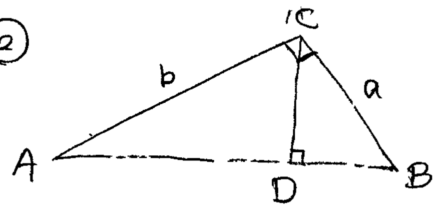
① 

we can see that

$c^2 + \frac{1}{2} \cdot a \cdot b \cdot 4 = (a+b)^2$

Therefore, $c^2 = a^2 + b^2$

10

② 

let $\overline{AC} = b$, $\overline{BC} = a$, $\overline{AB} = c$

$\angle ACB = 90°$

Since ACD, BCD, ABC are similar to each other, there is a constant k, independent to the choice of triangles that

$S_{\triangle ACD} = b^2 \cdot k$ $\quad S_{\triangle ABC} = c^2 \cdot k$

$S_{\triangle BCD} = a^2 \cdot k$

Note that $k \neq 0$.

Since $S_{\triangle ABC} = S_{\triangle ACD} + S_{\triangle BCD}$

Then $c^2 = a^2 + b^2$

Notice that $S_{\triangle ABC}$ represents the area of triangle ABC

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

Assume that $\sqrt[7]{3}$ is rational.

$\exists\, m, n \in \mathbb{Z}$ such that

$\sqrt[7]{3} = \frac{m}{n} \Rightarrow 3 = \frac{m^7}{n^7}$

Since $\exists\, a, b \in \mathbb{Z}$ such that

$m = a \cdot 3^i$ for $i \geq 0$, $i \in \mathbb{Z}$

$n = b \cdot 3^j$ for $j \geq 0$, $j \in \mathbb{Z}$

Note that a, b are NOT divisible by 3.

So since $m^7 = 3n^7$.

$\Rightarrow a^7 \cdot 3^{7i} = b^7 \cdot 3^{7j+1}$ This is impossible!

why

9

So $\sqrt[7]{3}$ is NOT a rational number.

So $\sqrt[7]{3}$ is irrational.

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

Mandelbrot set is a set of points satisfying the mapping $z = z^2 + r$

0

(b) (5 points) Define the Feigenbaum constant. Explain everything!

Feigenbaum constant $c = \lim_{n \to \infty} \frac{r_{n+1} - r_n}{r_n - r_{n-1}} = 4.669\cdots$

3

Every iteration, you have a r_i ($i \in \mathbb{N}$). It was discovered by Feigenbaum and named after it.

4. (10 pts. total)
(a) (6 pts.) For any position P in the (n^2-1)-puzzle, let $[i, j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1, 2, 3, \ldots, n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + \mathit{inv}\,(\pi(P)) \quad ,$$

where $\mathit{inv}\,(\pi)$ is the number of inversions of the permutation π.
Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd.**
Note: You may use the lemma that if you exchange *any* **two** elements in a permutation, the number of inversions *always* changes by an odd integer.

proof: According to the lemma, if you exchange any two elements in a permutation, the number of inversions always changes by an odd integer. Let the location of blank in P be (i, j). Then the location of blank in Q would be either (i±1, j) or (i, j±1). Therefore, S(Q) preverses the same parity of S(P)

6

(b) (4 pts)
Let

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & \\ 6 & 7 & 8 \end{pmatrix} \quad .$$

Can you reach position ~~P~~ Q from position P, by a sequence of legal moves? Explain!

According to (a), $S(P) = 2+2+\mathit{inv}(\pi(P)) = 4+1 = 5$
$S(Q) = 2+3+\mathit{inv}(\pi(Q)) = 2+3+4 = 9$

4

Yes, Q is reachable from P, since S(Q) and S(P) have the same parity.

5. (10 points)
Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

6

Cauchy's integral theorem is $f(z_0) = \frac{1}{2\pi i}\int_C \frac{f(\xi)}{z-\xi}\, d\xi$

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x, y) + iv(x, y)$ where $u(x, y), v(x, y)$ satisfy the above system of two equations?

~~Cauchy's integral theorem~~
Chaucy-Riemann equation!
The function $u(x,y) + iv(x,y)$ can be expanded around each $(x,y) = (x_0, y_0)$ in a series convergent in a circle passing through the singular point nearest to (x_0, y_0)

5

7. (10 points) Who discovered the quaternions? What city did that person live in?

Hamilton. He lived in Dublin

10

8. (10 points) What is Heron's formula, what century did Heron live in?

$A = \sqrt{s(s-a)(s-b)(s-c)}$

A : area of triangle
a, b, c : three edges of the triangle
$s = \frac{1}{2}(a+b+c)$

He lived in 2nd century BC 1st AD

7

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

Isaac Newton studied calculus. His teacher was Isaac Barrow.
He didn't care much about vigority of proofs.
Newton's position was a priest after leaving Cambridge.

3

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

Leibnitz was born in Leipzig. He spent most of his life in France.

3

11. (10 points total)
(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$$\frac{2}{\pi} = \cos\frac{\pi}{4} \cdot \cos\frac{\pi}{8} \cdot \cos\frac{\pi}{16} \cdot \cos\frac{\pi}{32} \cdots$$

10

(b) (5 points) State the names of two people who initiated the use of logarithms

John Neper, Briggs

NAME: (print!) Ji Xu

E-Mail address: xj4714970008@gmail.com

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

---

1. 10 (out of 10)

2. 0 (out of 10)

3. 7 (out of 10)

4. 8 (out of 10)

5. 4 (out of 10)

6. 10 (out of 10)

7. 0 (out of 10)

8. 10 (out of 10)

9. 8 (out of 10)

10. 7 (out of 10)

11. 9 (out of 10)

---

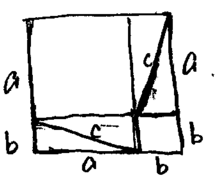
total: 73 (out of 110)

$\cdot 1.134 = 83$

10

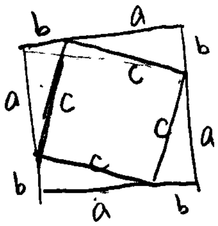
1. (10 pts.) Give two proofs of the Pythagorean theorem. $a^2 + b^2 = c^2$.

(1)



$(a+b)^2 = a^2 + 2ab + b^2$.

From the graph above, we can take ~~4~~ pieces of [triangle with legs b, a] to make the graph below.

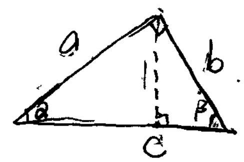


$(a+b)^2 = c^2 + 2ab$.

therefore, $a^2 + 2ab + b^2 = c^2 + 2ab$.

$$a^2 + b^2 = c^2.$$

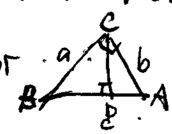
(2)



$\alpha + \beta = 90°$.

For every similar triangles whose ~~largest side~~ is a. there is a constant that area $= ka^2$.

so for



area of $\triangle ABC = kc^2$

area of $\triangle BCD = ka^2$. area of $\triangle ACD = kb^2$

so $kc^2 = ka^2 + kb^2$.

$$c^2 = a^2 + b^2.$$

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

let $\sqrt[7]{3} = \frac{m}{n}$. m and n are positive integars.

$m = \sqrt[7]{3}\, n$.

$m^7 = 3n^7$

For positive integar m and n. we can write

$n = 2^i m$. $i \geq 0$.

$(2^i m)^7 \neq 3(2^j n)^7$

$2^{7i} m^7 = 3 \cdot 2^{7j} n^7$

$2^{7i} m^7 = (2 \cdot 2^{7j} + 2^{7j}) n^7 = (2^{7j+1} + 2^{7j}) n^7$

$2^{7i} \neq 2^{7j+1} + 2^{7j}$

So this is contradiction. there is no Integar m and n for $\frac{m}{n} = \sqrt[7]{3}$.

so $\sqrt[7]{3}$ is irrational.

0

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

The Mandelbrot set is the set of complex numbers c for which the function $f_c(z) = z^2 + c$ does not diverge. when iterated from $z = 0$.

5

(b) (5 points) Define the Feigenbaum constant. Explain everything!

The Feigenbaum constant are ~~two~~ a mathmatical ~~constants~~ which both express ratios in a bifuration diagram for a non-linear map.

more details

2

4. (10 pts. total)
(a) (6 pts.) For any position P in the $(n^2$-1)-puzzle, let $[i, j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1, 2, 3, \ldots, n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + \mathit{inv}\,(\pi(P)) \quad ,$$

where $\mathit{inv}\,(\pi)$ is the number of inversions of the permutation π.
Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd**.
Note: You may use the lemma that if you exchange *any* **two** elements in a permutation, the number of inversions *always* changes by an odd integer.

for every move exchanges the blank with a neighboring.
$inv(\pi)$ changes by an odd number and $i+j$ changes by ± 1.

You did not finish!

4

(b) (4 pts)
Let

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & \\ 6 & 7 & 8 \end{pmatrix} \quad .$$

Can you reach position Q from position P, by a sequence of legal moves? Explain!

let the blank be 9.

$\therefore \pi_P = (1\,2\,3\,5\,9\,4\,6\,7\,8)$ $\quad \pi_Q = (4\,2\,1\,3\,5\,9\,6\,7\,8)$.

$inv(\pi_P) = 1+4 = 5.$ $\quad inv(\pi_Q) = 3+4 = 7.$

$S(P) = inv(\pi_P) + 2+2.$ $\quad S(Q) = inv(\pi_Q) + 2+3.$

$= 5+2+2$ $\quad = 7+2+3$

$= 9.$ $\quad = 12.$

$S(P)$ is odd, $S(Q)$ is even. so P cannot reach Q.

4

5. (10 points)
Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

4

let $|H| = m$. $|G| = n$.

$H = \{h_1, h_2, h_3, \cdots, h_m\}$.

$\{ah_1, ah_2, ah_3, \cdots, ah_m\}$ has no overlap with H.

$gh_i = gh_j$. $i \neq j$.

$(g^{-1}g)(h_i) = (g^{-1}g)(h_j)$

$eh_i = eh_j$

$h_i = h_j$.

You did not finish

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x,y)+iv(x,y)$ where $u(x,y), v(x,y)$ satisfy the above system of two equations?

Name : Cauchy - Riemann Equations.

the whole function is analytic on a domain.

10

7. (10 points) Who discovered the quaternions? What city did that person live in?

Jean le Rond d'Alembert.

Pairs. France.

0

8. (10 points) What is Heron's formula, what century did Heron live in?

Area of triangle, $A = \sqrt{s(s-a)(s-b)(s-c)}$

~~first~~ century.

10

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

He study in Trinity College.

teaher: Isaac Barow.

After he left Cambrige, he served as Warden (1696-1700) and Master (1700-1727) of the Royal Mint, as well as president of the Royal Society (1703-1727).

8

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

City: Leipzing He spend most of his life in ~~German~~ Hanover

king GeorgeI.

7

11. (10 points total)
(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdot \cdots$$

(b) (5 points) State the names of two people who initiated the use of logarithms

John Napier, Jost Burgi.

9

/

NAME: (print!) Alicia Sukhram

E-Mail address: asukhram1@gmail.com

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

1. 10 (out of 10)
2. 5 (out of 10)
3. 7 (out of 10)
4. 10 (out of 10)
5. 5 (out of 10)
6. 0 (out of 10)
7. 10 (out of 10)
8. 0 (out of 10)
9. 10 (out of 10)
10. 7 (out of 10)
11. 7 (out of 10)

total: (out of 110)
$71 \cdot 1.134 = 80.5$

5. Let H have m elements
Let G have r elements that are not in H.

$H = \{h_1, \ldots, h_m\}$.

$\{a_1h_1, \ldots, a_1h_m\}$ is a coset where each element in this set is different and there is no overlap with H.

Proof: suppose there are two elements that are the same in the coset:

$a_1h_i = a_1h_j \quad i \neq j$.

Then $a_1^{-1}(a_1h_i) = a_1^{-1}(a_1h_j)$

$(a_1^{-1}a_1)h_i = (a_1^{-1}a_1)h_j$

$e h_i = e h_j$

$h_i = h_j$

contradiction. Therefore, no two elements are the same.

Proof: Suppose there is overlap between coset and H.

$a_1h_i = h_j$

$a_1^{-1}(a_1h_i) = a_1^{-1}h_j$

$(a_1^{-1}a_1)h_i = a_1^{-1}h_j$

$e h_i = a_1^{-1}h_j$

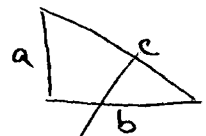
$h_i = a_1^{-1}h_j$

But $a_1 \notin H$ and $a_1^{-1} \in H$.
contradiction, Therefore, no overlap

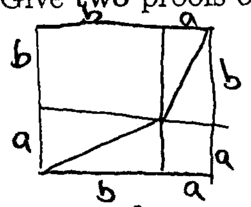
we can continue forming cosets
$\{a_1h_1, \ldots, a_1h_m\}$
$\{a_2h_1, \ldots, a_2h_m\}$
$\vdots$
$\{a_r h_1, \ldots, a_r h_m\}$

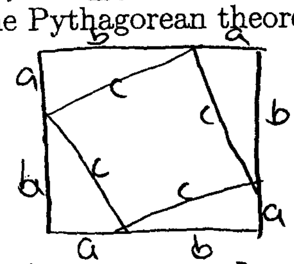
Then we have all the elements in G, so $|G| = r * m$
Therefore $\frac{|G|}{|H|} = \frac{r*m}{m} = r \in \mathbb{Z}$

1. (10 pts.) Give two proofs of the Pythagorean theorem.



I. Consider:





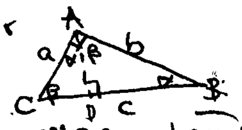
area $= (a+b)^2$
$= b^2 + a^2 + 4(\frac{1}{2}ab)$

area $= (a+b)^2$
$= c^2 + 4(\frac{1}{2}ab)$

These two areas are equal: $b^2 + a^2 + \cancel{4(\frac{1}{2}ab)} = c^2 \cancel{+4(\frac{1}{2}ab)}$
thus, $b^2 + a^2 = c^2$.

10

II. Consider



$\alpha + \beta = 90$. Triangles ABC, ABD, ADC are similar.

Since ΔABD and ΔADC make up ΔABC,
$ka^2 + kb^2 = kc^2$
$\Rightarrow a^2 + b^2 = c^2$

By Lemma, $\Delta ABC = k c^2$
$\Delta ABD = k b^2$
$\Delta ADC = k a^2$

$\exists k \in \mathbb{Z}$ such that longest side squared is same for similar Δ

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

By contradiction, assume $\sqrt[7]{3}$ is rational

Then $\exists m, n \in \mathbb{Z}$, $n \neq 0$ s.t. $\sqrt[7]{3} = \frac{m}{n}$, (m and n have no common factors)

$3 = \frac{m^7}{n^7} \rightarrow 3n^7 = m^7$.

Let $m = 3^i a$ and $n = 3^j b$, where $i, j \geq 0$ and a, b are not divisible by 3

Then $3(3^j b)^7 = (3^i a)^7 \rightarrow 3^{7j+1} b = 3^{7i} a^7$

This is impossible. These two quantities cannot be equal because one side will have even power with 3 and other side will have odd. Therefore, $\sqrt[7]{3}$ is irrational.

5

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

c such that $z_n \to z_{n-1}^2 + c$ converges to a finite orbit

5

(b) (5 points) Define the Feigenbaum constant. Explain everything!

$x_{n+1} = r x_n (1 - x_n)$.

$\lim_{n \to \infty} \frac{r_n - r_{n-1}}{r_{n+1} - r_n} = 4.669\ldots$ which is a limit on population

what is r_n?

2

4. (10 pts. total)
(a) (6 pts.) For any position P in the (n^2-1)-puzzle, let $[i, j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1, 2, 3, \ldots, n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + \mathit{inv}\,(\pi(P)) \quad ,$$

where $\mathit{inv}\,(\pi)$ is the number of inversions of the permutation π.
Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd.**
Note: You may use the lemma that if you exchange *any* **two** elements in a permutation, the number of inversions *always* changes by an odd integer.

Case 1, horizontal move
The blank is exchanged with element to ~~the~~ left or right which changes the parity of the number of inversions
The column number of the blank changes by 1 which changes the parity. The Parity changes twice, so it is the same as the original parity.

6

Case 2, vertical move.
The blank is exchanged with element to the top or bottom which changes the parity
The row number of the blank changes by 1 which changes the parity. The parity changes twice, so it is the same ~~as the~~ original parity

In either case the parity remains the same after 1 move. Therefore after several moves, the parity will be the same

(b) (4 pts)
Let

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & \\ 6 & 7 & 8 \end{pmatrix}$$

Can you reach position ~~P~~ Q from position P, by a sequence of legal moves? Explain!

4

$\pi(P) = 1\ 2\ 3\ \underset{(1)}{5}\ \underset{(4)}{9}\ 4\ 6\ 7\ 8 \quad inv(\pi) = 5 \quad S(P) = 2+2+5 = 9.$

$\pi(Q) = \underset{(3)}{4}\ \underset{(1)}{2}\ 1\ 3\ 5\ \underset{(3)}{9}\ 6\ 7\ 8 \quad inv(\pi) = 7 \quad S(Q) = 2+3+7 = 12.$

Parity changes from odd to even. Going from P to Q is impossible.

5. (10 points)
Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

10 See front page

mi4

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x,y) + iv(x,y)$ where $u(x,y), v(x,y)$ satisfy the above system of two equations?

equation of dynamics: Hamilton-Jacobi equation,
establish relation between dynamics and contact transformations

0

7. (10 points) Who discovered the quaternions? What city did that person live in?

Hamilton

lived in Dublin

10

8. (10 points) What is Heron's formula, what century did Heron live in?

$\sqrt{a \pm \sqrt{b}}$

17th century

0

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

studied at Cambridge, fellow of Trinity College
Isaac Barrow was his teacher.
Barrow appointed Newton for Barrow's position, wrote geometrical lectures.
After cambridge, he was appointed Lucasian professor
in cambridge until 1696 as warden, later master of mint

used by Leibniz

10

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

Leipzig
Spent most of life in Germany
Huygens ~~Frederick~~?

7

11. (10 points total)

(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$\frac{2}{\pi} = \cos\frac{\pi}{4} \cos\frac{\pi}{8} \cos\frac{\pi}{16} \cos\frac{\pi}{32} \cdots$

(b) (5 points) State the names of two people who initiated the use of logarithms

Stevin

John Napie

7

/

NAME: (print!) LYNNE RICHMAN

E-Mail address: " . " @gmail.com

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

---

1. 5 (out of 10)

2. 5 (out of 10)

3. 7 (out of 10)

4. 10 (out of 10)

5. 10 (out of 10)

6. 10 (out of 10)

7. 0 (out of 10)

8. 10 (out of 10)

9. 0 (out of 10)

10. 0 (out of 10)

11. 10 (out of 10)

total: (out of 110)

$67 \cdot 1.134 = 76$

you can do better than that!

5. Let $G = \{g_1, \dots, g_n\}$, $|G| = n$.
Let $H = \{h_1, \dots, h_m\}$, $|H| = m$.

Case I: $H = G$. Then $n = m$, so $\frac{|G|}{|H|} = \frac{n}{m} = 1$.

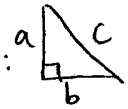
Case II: $H \neq G$. Then $\exists g_i \in G$ s.t. $g_i \notin H$. Look at $g_iH = \{g_ih_1, \dots, g_ih_m\}$.

Claim: Each element of g_iH is different.
Pf: Assume not. Then $\exists g_ih_i, g_ih_j$ s.t. $g_ih_i = g_ih_j$. So $g_i^{-1}g_ih_i = g_i^{-1}g_ih_j$. Thus $eh_i = eh_j$. Thus $h_i = h_j$. This is a contradiction.

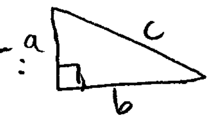
Claim: There is no overlap between H and g_iH.
Pf: Assume not. Then $\exists h_i \in H$ and $g_ih_j \in g_iH$ s.t. $h_i = g_ih_j$. Then $h_j^{-1}h_i = g_ih_jh_j^{-1}$. So $h_jh_i^{-1} = g_ie$. Since H is a group, $h_jh_i^{-1} \in H$. This is a contradiction, because $g_i \notin H$.

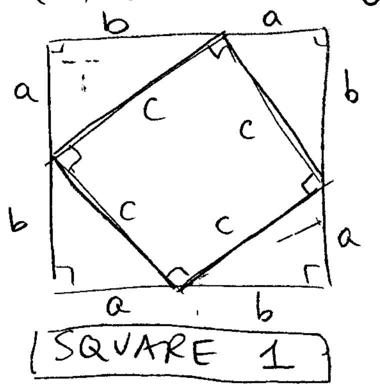
If $g_iH \cup H$ doesn't cover G, look at g_jH, for $g_j \in G$, $g_j \notin H$, $g_i \neq g_j$.

Claim: No overlap between g_iH and g_jH. Pf: Assume not. Then $\exists h_i, h_j$ s.t.

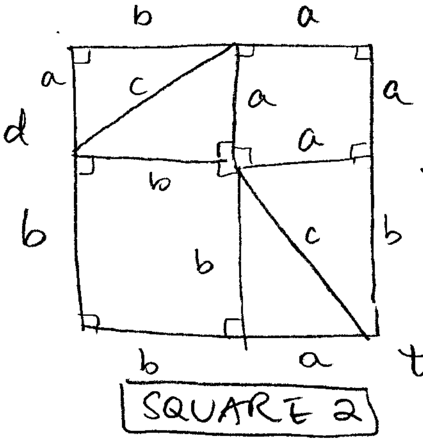
1. (10 pts.) Give **two** proofs of the Pythagorean theorem. :  $a^2+b^2=c^2$

(1) LOOK at:

Let T: 



and



(5)

. Clearly the area of the first square is $(a+b)^2$, which is also the area of square 2.

Since $A(\text{square } 1) = (a+b)^2 = c^2 + 4\cdot A(T)$ and $A(\text{square } 2) = (a+b)^2 = a^2+b^2+4\cdot A(T)$, and $A(\text{square } 1) = A(\text{square } 2)$; then $c^2+4\cdot A(T) = a^2+b^2+4\cdot A(T)$. Thus $c^2=a^2+b^2$.

(2) Don't remember the second proof using similar triangles. ⍨

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

(odd and even are irrelevant)

Assume it's rational. Then $\exists n,m \in \mathbb{Z}$ s.t. $(3)^{1/7} = \frac{n}{m}$. So $3 = \frac{n^7}{m^7}$ and $3m^7=n^7$. Let $n = 3^i\cdot a$, $m = 3^j \cdot b$, s.t. $i,j \in \{0,1,2,\dots\}$ and $3 \nmid a$ and $3 \nmid b$. Thus $3(3^j\cdot b)^7 = (3^i\cdot a)^7$, so $3(3^{7j}\cdot b^7) = 3^{7i}\cdot a^7$. Therefore $3^{7j+1}\cdot b^7 = 3^{7i}\cdot a^7$. Since $3\nmid a$ and $3 \nmid b$, thus $3\nmid a^7$ and $3\nmid b^7$. Thus we have reached a contradiction as it ~~to~~ impossible to have a number which is the product of an even power of 3 and a number not divisible by 3 AND also an odd power of 3 and a number not divisible by 3.

(5)

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

$S = \{ c : z \to z^2 + c \text{ does not diverge}\}$

$f_z(z) = z_{n-1}^2 + c$, $z_0 = 0$.

(5)

(b) (5 points) Define the Feigenbaum constant. Explain everything! $f(x) = rx(x-1)$, $x \in (0,1)$

(2) $\lim\limits_{n\to\infty} \frac{r_n - r_{n-1}}{r_{n+1}-r_n} = 4.6\dots$, where r_i is the ~~Cutoff~~ doubling period of $f(x)$ (so that $f(x)$ converges to double the number of points.)

4. (10 pts. total)
(a) (6 pts.) For any position P in the (n^2-1)-puzzle, let $[i,j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1,2,3,\ldots,n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

10

$$S(P) = i + j + \ inv\,(\pi(P)) \quad ,$$

where $inv\,(\pi)$ is the number of inversions of the permutation π.
Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd**.
Note: You may use the lemma that if you exchange *any* **two** elements in a permutation, the number of inversions *always* changes by an odd integer.

Case I: Perform horizontal slide to get to position Q. Then i does not change and j changes by 1 (either positive or negative). By lemma stated above, the # of inversion changes by an odd integer, 2n+1. Thus $S(Q) = S(P) \pm 1 + 2n{+}1$
So S(Q) has same parity as S(P).
Case II: Perform vertical slide, to go to position Q. Then i changes by ±1, j doesn't change, $inv(\pi(Q)) = inv(\pi(P)) + 2n{+}1$
So $S(Q) = S(P) \pm 1 + 2n{+}1$. So S(Q) has the same parity as S(P).

(b) (4 pts)
Let

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & \\ 6 & 7 & 8 \end{pmatrix} \quad .$$

Can you reach position Q from position P, by a sequence of legal moves? Explain!

$S(P) = 2 + 2 + 1 + 4 = 9$

$S(Q) = 2 + 3 + 3 + 1 + 3 = 12$

Since S(P) is odd and S(Q) is even, there is no way to reach Q from P using legal sliding moves; as proved in (a).

5. (10 points)
Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

10

a)
6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

b)
What is special about the function $u(x,y)+iv(x,y)$ where $u(x,y), v(x,y)$ satisfy the above system of two equations?

a) Cauchy-Riemann equations.
b) Definition of a complex function, defined by Riemann. (1851)

10

7. (10 points) Who discovered the quaternions? What city did that person live in?

Couldn't find this in my notes :(0

8. (10 points) What is Heron's formula, what century did Heron live in?

$A = \sqrt{s(s-a)(s-b)(s-c)}$, lived in ~62 AD 10

(Area of a triangle)

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

Don't know, not in my notes :(

0

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

Not in my notes :(0

11. (10 points total)
(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$$\frac{2}{\pi} = \cos\left(\frac{\pi}{4}\right) \cdot \cos\left(\frac{\pi}{8}\right) \cdot \cos\left(\frac{\pi}{16}\right) \cdot \ldots$$

10

(b) (5 points) State the names of two people who initiated the use of logarithms

John Napier & Henry Briggs

NAME: (print!) POORVA SAMPAT

E-Mail address: poorva.sampat@rutgers.edu

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

1. 10	(out of 10)
2. 5	(out of 10)
3. 9	(out of 10)
4. 10	(out of 10)
5. 5	(out of 10)
6. 0	(out of 10)
7. 10	(out of 10)
8. 7	(out of 10)
9. 0	(out of 10)
10. 0	(out of 10)
11. 10	(out of 10)

total: 66 (out of 110)

$1.134 = 75$

(5) Let $G = \{g_1, g_2, g_3, \ldots, g_n\}$

If H is a subgroup, then

$H = \{h_1, h_2, h_3 \ldots h_m\}$

such that $h_1, h_2 \ldots h_m \in G$

For some $g_i \in G$

$g_i H = \{g_i h_1, g_i h_2, g_i h_3 \ldots g_i h_m\}$ but $g_i h_1 \ldots \notin H$

such that $g_i h_1, g_i h_2 \ldots g_i h_m \in G$

because of closure property of groups.

but $g_i H$ is a new subgroup with same number of element

$H_1 = \{g_i h_1, g_i h_2 \ldots g_i h_m\}$

Similarly n other such subgroups can be created

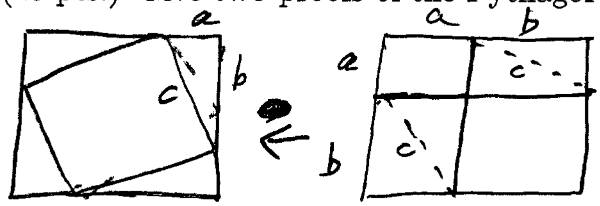
$G = H \cup H_1 \ldots$

$|G| = |H| n \Rightarrow \frac{|G|}{|H|}$ is an integer

$g_i H$ is not a subgroup, it is a coset

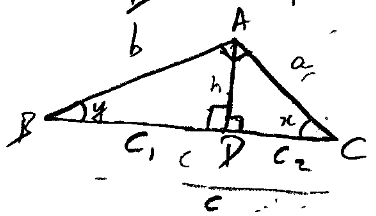
1. (10 pts.) Give two proofs of the Pythagorean theorem.

10



If we cut the two squares & the 4 Δ's in the second picture & place it on the first it is a perfect fit

$\cancel{4\Delta's} + a^2+b^2 = \cancel{4\Delta's} + c^2 \Rightarrow a^2+b^2=c^2$



We see that

$\triangle ABC \sim \triangle ADC$ b/c $\angle CAB = \angle ADC = 90°$

$\angle ACD = \angle ACD$ call it x

$\angle DAC = \angle ABC = 90° - x$

So, $\frac{c_2}{a} = \frac{a}{c} \Rightarrow a^2 = c_2 c$

$\frac{c_1}{b} = \frac{b}{c} \Rightarrow b^2 = c_1 c$

Also $\triangle ABC \sim \triangle ADB$

b/c $\angle CAB = \angle ADB = 90°$

$\angle ABD = \angle ABD = y$

$\angle ACB = \angle DAB = 90° - y$

$a^2+b^2 = c_2 c + c_1 c = c^2$

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

Assume $\sqrt[7]{3}$ is rational $\Rightarrow \sqrt[7]{3} = \frac{m}{n}$ where gcd$(m,n)=1$ & $n \neq 0$

$\Rightarrow 3n^7 = m^7$

Lemma:

Every number $\bar{n}$ can be written uniquely as $\bar{n} = 3^i \bar{m}$

So let $m^7 = 3^i \bar{m}$

$3n^7 = 3^{j+1} \bar{m}$ i, j are unique b/c of lemma

$3^i \bar{m} = 3^{j+1} \bar{m}$ But $i \neq j+1$

This is not possible

Hence proved by contradiction.

not clear

5

Write a comp. prog

i = 0

while $\bar{n} > 0$ & $\bar{n}$ mod $3 = 0$

$\bar{n} = \bar{n}/3$

i++

print (3 ** i) * $\bar{n}$

exp

$\bar{m}$ after loop

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

$z_0 = 0$, $z_n = z_{n-1}^2 + c$

c is in Mandelbrot set when the above equation converges

5

(b) (5 points) Define the Feigenbaum constant. Explain everything!

Feigenbaum constant is the ratio b/w the distances of bifurcation of limiting points in the function

$$\lim_{n\to\infty} \frac{a_{n-1} - a_{n-2}}{a_n - a_{n-1}}$$

$\lambda = 4.996$

what is a_n?

4

4. (10 pts. total)

(a) (6 pts.) For any position P in the $(n^2\text{-}1)$-puzzle, let $[i, j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1, 2, 3, \ldots, n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + \mathit{inv}\,(\pi(P)) \quad ,$$

where $\mathit{inv}\,(\pi)$ is the number of inversions of the permutation π.

Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd**.

Note: You may use the lemma that if you exchange *any* **two** elements in a permutation, the number of inversions *always* changes by an odd integer.

10

There are two types of legal moves allowed -

i) horizontally - if we move horizontally then j will change. But for each change in j, there will be an equal change in $inv(\pi(P))$ by the lemma as exchanging any two elements causes # of inv to change by an odd integer. Thus, the change will always be an even number.

ii) Vertically - if we move vertically then i will change. But for each change in i, there will be an equal change in the # of inv. by lemma. Thus, the final change is always an even amount.

If $S(P)$ is even to begin with then adding/subtracting an even amount* will lead to an even $S(Q)$. Similarly, if $S(P)$ begins as odd, $S(Q)$ will also be odd.

*as allowed by the above two legal moves

(b) (4 pts)

Let

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & \\ 6 & 7 & 8 \end{pmatrix} \quad .$$

Can you reach position Q from position P, by a sequence of legal moves? Explain!

$S(P) = 2 + 2 + 5 = 9$

$S(Q) = 2 + 3 + 7 = 12$

No, it is not possible to reach Q from position P as the parity of $S(P)$ is different from the parity of $S(Q)$. odd vs even.

5. (10 points)

Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

incomplete and partially wrong 5

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x,y) + iv(x,y)$ where $u(x,y), v(x,y)$ satisfy the above system of two equations?

~~Hamilton Jacobi eq.~~ polar co-ordinates?

Create a circle ellipsoid or graph

0

7. (10 points) Who discovered the quaternions? What city did that person live in?

Hamilton. Lived in Dublin.

10

8. (10 points) What is Heron's formula, what century did Heron live in?

$A = \sqrt{s(s-a)(s-b)(s-c)}$ C 2nd BC

7

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

Center for scientific exchange – European university

0

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

Paris

In France

Gtkl

The King reigning during 1693

0

11. (10 points total)

(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$$\frac{2}{\pi} = \cos\frac{\pi}{4} \cos\frac{\pi}{8} \cos\frac{\pi}{16} \cos\frac{\pi}{32} \cdots$$

10

(b) (5 points) State the names of two people who initiated the use of logarithms

John Neper , Briggs

NAME: (print!) Jing Yang

E-Mail address: JY410@scarletmail.rutgers.

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

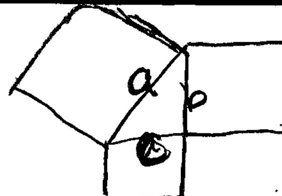
---

1. 5 (out of 10)
2. 2 (out of 10)
3. 4 (out of 10)
4. 10 (out of 10)
5. 2 (out of 10)
6. 0 (out of 10)
7. 10 (out of 10)
8. 10 (out of 10)
9. 5 (out of 10)
10. 7 (out of 10)
11. 10 (out of 10)

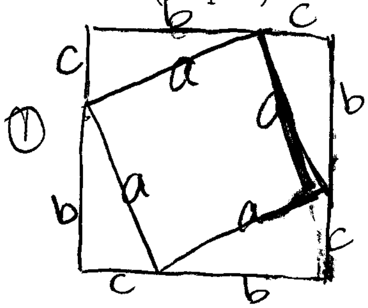
---

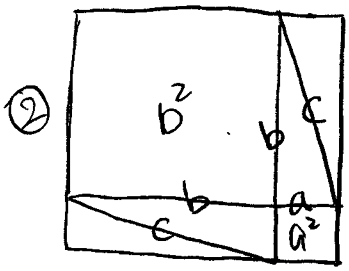
total: (out of 110)

$64 \cdot 1.134 = 73$



1. (10 pts.) Give two proofs of the Pythagorean theorem.





We can prove that $a^2 = b^2 + c^2$

For image ① we get that. for a triangle

and to make four identity triangle. and

set together, we get $a^2 = b^2 + c^2$

if we arrange differently like image ②

we get b^2 and a^2 also by 4 identity triangle.

5

2nd proof?

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

$\sqrt[7]{3} = \frac{m}{n} \Rightarrow$ $3 = \frac{m^7}{n^7}$ ①

(m, n is divisible by 3, until they can't divisible by 3).

by the lemma let $n = 3^i m$ (m not divisible by 3, $i \in \mathbb{Z}^+$)

$n^7 = 3^{7i} m$

plug in back to ①.

$m^7 = 3n^7$

$= 3 \cdot 3^{7i} m = 3^{7i+1} m$.

$3n^7 = 3^{7i+1} m$

since n^7 is alway odd number, but 3^{7i+1} is even exponet then it's contradiction.

2

does not make sense

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

Z_0, $Z_{n+1} = Z_n^2 + C$.

$Z_0 = 0$.

2

(b) (5 points) Define the Feigenbaum constant. Explain everything!

$\delta = \lim_{n \to \infty} \frac{a_{n+2} - a_{n+1}}{a_{n+1} - a_n} = 4.66\cdots$ The distance between two point are keeping decreasing in the constant.

What is a_n?

2

4. (10 pts. total)
(a) (6 pts.) For any position P in the (n^2-1)-puzzle, let $[i, j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1, 2, 3, \ldots, n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + \mathit{inv}\,(\pi(P)) \quad ,$$

where $\mathit{inv}\,(\pi)$ is the number of inversions of the permutation π.
Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd**.
Note: You may use the lemma that if you exchange *any* **two** elements in a permutation, the number of inversions *always* changes by an odd integer.

"permutation" + "the row #" "+" column # "Invension" + " the taxi cab.
In any single legal move. n^2 get's exchanged with other int.
but all the other ints stay the same, the # of invension change by an odd number, but taxi-cab distance change by ± 1,
Change row # or col # by exactly one.
Therefore parity of the sum is the same every time to make the legal move.

6

(b) (4 pts).
Let

$i=2, j=2$

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & \boxed{9} & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad P' = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & \boxed{9} \\ 6 & 7 & 8 \end{pmatrix} \quad .$$

$i=2, j=3$

4

Can you reach position P' from position P, by a sequence of legal moves? Explain!

$P(\pi) = 1, 2, 3, 5, 9, 4, 6, 7, 8$
$\mathrm{inv}(\pi) = 0+0+0+1+4+0+0+0+0$
$= 5$
$S(P) = \mathrm{inv}(\pi) + i + j = 9$

$P'(\pi) = 4, 2, 1, 3, 5, 9, 6, 7, 8.$
$\mathrm{inv}(\pi) = 3+1+0+0+0+3 = 7$
$S(P') = \mathrm{inv}(\pi) + i + j = 7+2+3 = 12.$

It's impossible that P to reach P', by the definition they are parity one is odd, one is even.

5. (10 points)
Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

2

$|H| = \{h_1, h_2, \cdots h_m\}$
$|G| = \{g_1, g_2, \cdots, g_n\}$

$\{g_2h_1, g_2h_2, g_2h_3, \cdots\}$ is distinct with H & G.
therefore.

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad ; \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x,y) + iv(x,y)$ where $u(x,y), v(x,y)$ satisfy the above system of two equations?

Laplace form.

X 0

7. (10 points) Who discovered the quaternions? What city did that person live in?

William Rowan Hamilton 1843

Dublin.

10

8. (10 points) What is Heron's formula, what century did Heron live in?

$A = \sqrt{s(s-a)(s-b)(s-c)}$; $s = \frac{a+b+c}{2}$.

Heron live in CE 60.

10

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

He study in University of Cambridge; Isaac Barrow;

5

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

He born in Leipzig ; Hanover ;

7

11. (10 points total)

(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdots$

(b) (5 points) State the names of two people who initiated the use of logarithms

John Napier ; Jost Burgi

10

/

NAME: (print!) SUSMITA PARUCHURI

E-Mail address: susmita.par@rutgers.edu

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

---

1. 6 (out of 10)
2. 5 (out of 10)
3. 2 (out of 10)
4. 10 (out of 10)
5. 5 (out of 10)
6. 6 (out of 10)
7. 10 (out of 10)
8. 10 (out of 10)
9. 0 (out of 10)
10. 0 (out of 10)
11. 10 (out of 10)

total: 64 (out of 110) $\cdot 1.134 =$ 73

$G = g_1, g_2 \dots g_m$

$H = h_1, h_2 \dots h_n$

$\exists\, h_i$ such that

$g_1 h_i, g_2 h_i, \dots g_m h_i \in G$

$g_1 h_1, g_2 h_1, g_3 h_1 \dots g_m h_1$

$g_1 h_2, g_2 h_2, g_3 h_2 \dots g_m h_2$

$g_1 h_3, g_2 h_3, g_3 h_3 \dots g_m h_3$

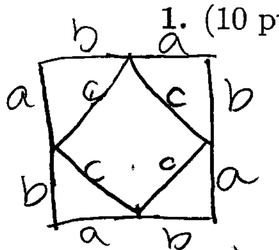
$\vdots$

$g_1 h_n, g_2 h_n, g_3 h_n \dots g_m h_n \in G$

$\frac{m}{n}$ must be an integer

not supposed to use this!

1. (10 pts.) Give two proofs of the Pythagorean theorem.

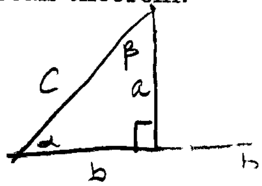


$(a+b)^2 = c^2 + 4(\tfrac{1}{2}ab)$

$a^2 + \cancel{2ab} + b^2 = c^2 + \cancel{2ab}$

$a^2+b^2=c^2$

5



$\alpha+\beta = 90°$

$\sin^2\alpha = \left(\frac{a}{c}\right)^2$

$\cos^2\alpha = \left(\frac{b}{c}\right)^2$

using $\sin^2\alpha + \cos^2\alpha = 1$

$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$

1

$\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1$

$\frac{a^2+b^2}{c^2} = 1 \rightarrow a^2+b^2=c^2$

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

$\sqrt[7]{3} = \frac{m}{n}$ m, n: nonzero positive integers

$3 = \frac{m^7}{n^7}$

$3n^7 = m^7$

set $n = 3^i a$, $m = 3^i b$

5

$3(3^i a)^7 = (3^i b)^7$

$3^{7i+1} a^7 = 3^{7i} b^7 \rightarrow 3^{7i+1}$ and 3^{7i} must be equal but the +1 gives $7i+1$ a different parity. This is impossible so $\sqrt[7]{3}$ cannot be written as $\frac{m}{n}$ → it is not rational.

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

$z \to z^2 + c$

0

(b) (5 points) Define the Feigenbaum constant. Explain everything!

$\lim_{n\to\infty} \frac{r_n - r_{n-1}}{r_{n+1} - r_n} = 4.667$ What is r?

2

4. (10 pts. total)

(a) (6 pts.) For any position P in the (n^2-1)-puzzle, let $[i, j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1, 2, 3, \ldots, n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + \ inv\,(\pi(P)) \quad ,$$

where $inv\,(\pi)$ is the number of inversions of the permutation π.

Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd**.

6 ☆ Note: You may use the lemma that if you exchange *any* **two** elements in a permutation, the number of inversions *always* changes by an odd integer.

~~Any~~ legal move is a horizontal or vertical switch.

Horizontal: According to given lemma, # of inversions changes by an odd integer. every horizontal move changes i by ± 1. So $S(Q)$ changes by an even #, which keeps it either odd or even depending on what it started with)

Vertical: Lemma: # inversions changes by odd, j is ± 1, so $S(Q)$ changes by an even number, preserving same parity.

(b) (4 pts)

Let

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & 9 & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & 9 \\ 6 & 7 & 8 \end{pmatrix} .$$

Can you reach position Q from position P, by a sequence of legal moves? ~~Explain!~~

$S(P) = inv(\pi(P)) + i + j = 1 + 4 + 2 + 2 = 9$ odd

$S(Q) = 3 + 1 + 3 + 3 + 2 = 4 + 8 = 12$ even

Different parities → this is impossible, proved in part (a)

4

5

5. (10 points)

Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

$|H| = n$, $|G| = m$, H is a subgroup of G

$G = g_1, g_2, g_3 \ldots g_m$ → continued on front.

$H = h_1, h_2, h_3 \ldots h_n$

not clear

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \ .$$

What is special about the function $u(x,y) + iv(x,y)$ where $u(x,y), v(x,y)$ satisfy the above system of two equations?

Cauchy - Riemann

6

7. (10 points) Who discovered the quaternions? What city did that person live in?

William Rowan Hamilton, Dublin

10

8. (10 points) What is Heron's formula, what century did Heron live in?

$A = \sqrt{s(s-a)(s-b)(s-c)}$ 1st cent. AD

10

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

Studied in France

Pupil of Galileo?

Professor

0

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

Born in France (Versailles?)

Lived in England

King Henry

0

11. (10 points total)

(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$\frac{2}{\pi} = \cos\frac{\pi}{4} \cos\frac{\pi}{8} \cos\frac{\pi}{16} \cos\frac{\pi}{32} \cdots$

10

(b) (5 points) State the names of two people who initiated the use of logarithms

John Neper (1614)

Henry Briggs (1624)

NAME: (print!) Xuehui Zhang

E-Mail address: xuehuizhang95@yahoo.com

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

---

1. 5 (out of 10)

2. 10 (out of 10)

3. 7 (out of 10)

4. 10 (out of 10)

5. 3 (out of 10)

6. 0 (out of 10)

7. 10 (out of 10)

8. 8 (out of 10)

9. 0 (out of 10)

10. 0 (out of 10)

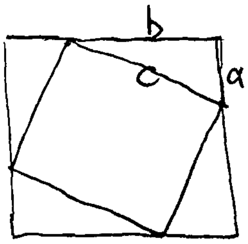
11. 9 (out of 10)

---

total: 64 (out of 110)

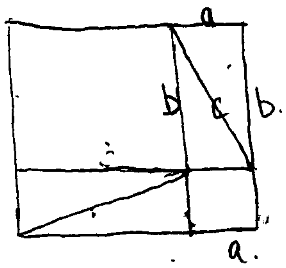
$64 \cdot 1.134 = 73$

1. (10 pts.) Give **two** proofs of the Pythagorean theorem.



$(a+b)^2 = c^2 + 4ab.$

5



$(a+b)^2 = a^2 + b^2 + 4ab.$

$c^2 + \cancel{4ab} = a^2 + b^2 + \cancel{4ab}$

$\Rightarrow a^2 + b^2 = c^2.$

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

Suppose $\sqrt[7]{3}$ is rational.

$\sqrt[7]{3} = \frac{m}{n}$, $\gcd(m,n) = 1$.

$\frac{m^7}{n^7} = 3.$

$m^7 = 3n^7$

Lemma: any number can be written in the form: $N = 3^i \cdot k$

$\Rightarrow m = 3^i \cdot p$, $\gcd(p,3) = 1$

$n = 3^j \cdot q$. $\gcd(q,3) = 1$

$(3^i \cdot p)^7 = 3(3^j \cdot q)^7$

$\therefore 3^{7i} p^7 = 3^{7j+1} q.$

$7i \neq 7j + 1.$

This is a contradiction

$\Rightarrow \sqrt[7]{3}$ is irrational.

10

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

Mandelbrot set is a set of numbers from function $f(z) = z^2 + c$, that has boundaries. namely, $f(0), f(f(0)), \cdots$ do not diverge

5

(b) (5 points) Define the Feigenbaum constant. Explain everything!

$$\lim_{n\to\infty} \frac{r_{n+1} - r_n}{r_n - r_{n-1}} = 4.669\ldots$$

Feigenbaum constant is a ~~are~~ ~~two~~ constant that express limit of ratios of bifurcation diagram for nonlinear map.

2

4. (10 pts. total)

(a) (6 pts.) For any position P in the (n^2-1)-puzzle, let $[i, j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1, 2, 3, \ldots, n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + \ inv\,(\pi(P)) \quad ,$$

where $inv\,(\pi)$ is the number of inversions of the permutation π.

Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd.**

Note: You may use the lemma that if you exchange *any* **two** elements in a permutation, the number of inversions *always* changes by an odd integer.

If you move n^2 vertically by one legal move the column-number doesn't change and the row-number changes by ± 1. If you move n^2 horizontally by one legal move, the row-number doesn't change and the column-number changes by ± 1. So whether you move n^2 vertically or horizontally, $i+j$ will change by ± 1.

6

At the same time, the number of inversions always changes by odd integers. So, $S(Q)$ is equal to $S(P) \pm 1 +$ an odd integer,

As a result, the parity of $S(P)$ = the parity of $S(Q)$

(b) (4 pts)

Let

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & \\ 6 & 7 & 8 \end{pmatrix} \quad .$$

Can you reach position P' from position P, by a sequence of legal moves? Explain!

$S(P) = 2+2+ inv(\pi(P)) = 4+5=9$

$inv(\pi(P)) = 1+4=5$

$S(P') = 3+2+ inv(\pi(Q)) = 5+7=12$

$inv(\pi(Q)) = 3+1+3=7$

You cannot reach position P' from P by legal moves. because the parity of $S(P) \neq$ the parity of $S(P')$

4

5. (10 points)

Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

$|H| = n \Leftarrow H = \{h_1, h_2, \cdots h_n\}$, $g_1H = \{g_1h_1, g_1h_2, \cdots g_1h_n\}$. $g_1 \in G$ $g_1H \in G$

$g_2H = \{g_2h_1, g_2h_2, \cdots g_2h_n\}$ $g_2 \in G$ $g_2H \in G$ $\Big\} m$

$|G| = mn$.

$\Rightarrow |G|/|H| = m$.

So, $|G|/|H|$ is always an integer

very incomplete!

3

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x,y)+iv(x,y)$ where $u(x,y), v(x,y)$ satisfy the above system of two equations?

the "canonical" form.

The equation helped him write the equations of dynamics.

0

7. (10 points) Who discovered the quaternions? What city did that person live in?

William Rowan Hamilton.

Dublin

10

8. (10 points) What is Heron's formula, what century did Heron live in?

"Heronic" formula for the area of a triangle.

$A = \sqrt{s(s-a)(s-b)(s-c)}$ in purely geometrical form — "Metrica".

First Century

10

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

Italy.

0

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

0

11. (10 points total)

(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$$\frac{2}{\pi} = \cos\frac{\pi}{4}\cos\frac{\pi}{8}\cos\frac{\pi}{16}\cos\frac{\pi}{32}\cdots$$

9

(b) (5 points) State the names of two people who initiated the use of logarithms

~~Viète~~, Henry Briggs, John Neper

/

NAME: (print!) Young . Jin Kim

E-Mail address: yk338 9

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

---

1. 10 (out of 10)
2. 5 (out of 10)
3. 5 (out of 10)
4. 4 (out of 10)
5. 5 (out of 10)
6. 0 (out of 10)
7. 10 (out of 10)
8. 10 (out of 10)
9. 0 (out of 10)
10. 4 (out of 10)
11. 10 (out of 10)

total: 63 (out of 110)

$63 \cdot 1.134 = 72$

#5 continued

Then: $g_2 h_i = g_2 h_j$

Hence $g_2^{-1}(g_2 h_i) = g_2^{-1}(g_2 h_j)$

leading. $h_i = h_j$.

Now let $g_2 h_i = h_j$.

Then $(g_2 h_i) h_i^{-1} = h_j h_i^{-1}$

$g_2 = h_j h_i^{-1} \in H$

Thus. $g_3 H = \{g_3 h_1, \cdots, g_3 h_m\}$

and we get $g_2 h_i = g_3 h_j$.

Repeat this n times and.

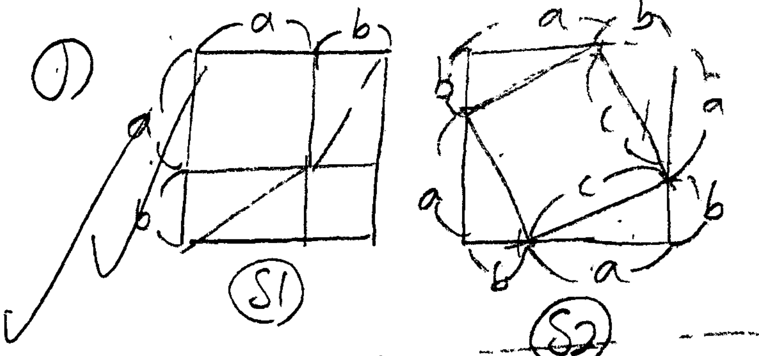
we get the desired result.

$\checkmark$

not clear

1. (10 pts.) Give two proofs of the Pythagorean theorem.

(1)



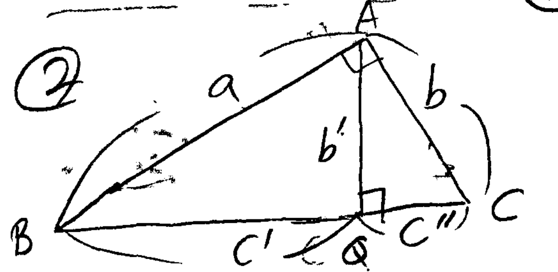
Suppose there is a perfect square with length $a+b$ for a side. Then the area is $(a+b)^2 = a^2+2ab+b^2$. The second square, which is of the same area, but different notation: c^2+2ab. Hence

$a^2+2ab+b^2 = c^2+2ab$

and $a^2+b^2=c^2$.

10

(2)



The triangle ABC, QAC, BQA are all similar to each other.

Then $\frac{c'}{a} = \frac{a}{c'+c''}$, $\frac{c''}{b} = \frac{b}{c'+c''}$ by ratio.

Then

$(c')^2 + c'c'' = a^2$

$(c'')^2 + c'c'' = b^2$

$(c'+c'')^2 = c^2$

$= a^2+b^2$.

Q.E.D

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

Suppose $\sqrt[7]{3}$ is rational and thus $\sqrt[7]{3} = \frac{m}{n}$

Then $3 = \frac{m^7}{n^7}$ and thus $m^7 = 3n^7$

Since m^7 is divisible by 3, we can say $m = 3^i a$ for some i and a. Similarly, $n = 3^j b$ for some j and b. Then $m^7 = 3n^7$

$= 3^{7i}a^7 = 3 \cdot 3^{7j} b^7 = 3^{7j+1} b^7$ Then since the exponent have different parity, this is not possible

5

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

Mandelbrot set is $z_{n+1} = z_n^2 + c$ where c is a constant and the values do not diverge as it grows.

3

not clear

(b) (5 points) Define the Feigenbaum constant. Explain everything!

Feigenbaum constants ~~are~~ is ~~two~~ one mathematical constants which both express ratios ~~...~~ bifurcation of a non-linear map. This is primarily used to predict population growth

2

4. (10 pts. total)
(a) (6 pts.) For any position P in the (n^2-1)-puzzle, let $[i, j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1, 2, 3, \ldots, n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + \mathit{inv}\,(\pi(P)) \quad ,$$

where $\mathit{inv}\,(\pi)$ is the number of inversions of the permutation π.
Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd**.
Note: You may use the lemma that if you exchange *any* **two** elements in a permutation, the number of inversions *always* changes by an odd integer.

0

(b) (4 pts)
Let

$$\begin{array}{c} i=2 \\ j=2 \end{array} \qquad \begin{array}{c} i=2 \\ j=3 \end{array}$$

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & \boxed{9} & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & \boxed{9} \\ 6 & 7 & 8 \end{pmatrix} \quad .$$

4

Can you reach position Q from position P, by a sequence of legal moves? Explain!

$S(P) = i + j + \mathit{inv}(\pi(P)) = 2 + 2 + (1+4) = 9$

$S(Q) = i + j + \mathit{inv}(\pi(Q)) = 2 + 3 + (3+1+3) = 12$

In order for the position change from P to Q be legal, the parity of $S(P)$ has to match with $S(Q)$. Since $S(P)$ is odd and $S(Q)$ is even, this cannot happen.

5. (10 points)
Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

5

If $H \subset G$, $\exists g \in G$ such that $g_2 \in H$.
Then $g_2 H = \{g_2 h_1, g_2 h_2, \ldots, g_2 h_m\}$

Continued on the first page.

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x,y) + iv(x,y)$ where $u(x,y), v(x,y)$ satisfy the above system of two equations?

0

Hamilton-Jacobi equation.

This is special because it is a characteristic function. (principal)

7. (10 points) Who discovered the quaternions? What city did that person live in?

William Rowan Hamilton, Dublin

10

8. (10 points) What is Heron's formula, what century did Heron live in?

$A = \sqrt{s(s-a)(s-b)(s-c)}$, 1st century

10

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

Paris, Christiaan Huygen,

Newton remained as a researcher.

0

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

~~Brunswick~~ Lepzig, London, Frederick the Great

4

11. (10 points total)

(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$$\frac{2}{\pi} = \cos\frac{\pi}{4} \cdot \cos\frac{\pi}{8} \cdot \cos\frac{\pi}{16} \cdot \cos\frac{\pi}{32}$$

(b) (5 points) State the names of two people who initiated the use of logarithms

John Neper, Henry Briggs

10

NAME: (print!) Maninder Gill

E-Mail address: mkg77@scarletmail.rutgers.edu

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

---

1. 5 (out of 10)
2. 9 (out of 10)
3. 7 (out of 10)
4. 7 (out of 10)
5. 3 (out of 10)
6. 0 (out of 10)
7. 10 (out of 10)
8. ~~10~~ (out of 10) 7
9. 5 (out of 10)
10. 7 (out of 10)
11. 2 (out of 10)

total: (out of 110)

$62 \cdot 1.134 =$

70.5

#5 $H \subset G$ $\exists g_2 \in G$ s.t $g_2 \notin H$

$g_2 H = \{g_2 h_1, g_2 h_2, \ldots g_2 h_m\}$

$g_2 h_i = g_2 h_j \therefore$

$g_2^{-1}(g_2 h_1) = (g_2^{-1} h_j)$

$h_i = h_j$

contradiction

$g_2 h_i = h_i$

$(g_2 h_i)(h_i^{-1}) = h_j h_i^{-1}$

$g_2(h_i h_i^{-1}) = h_j h_i^{-1}$

$g_2 h_i = g_3 h_j$

$g_3 = g_2 h_i h_j^{-1}$

contradiction

$\boxed{}$

very incomplete, incoherent

1. (10 pts.) Give two proofs of the Pythagorean theorem.

(1) Area of triangle 1 : $a^2+b^2+4(\text{triangles})=A$

Area of triangle 2: $c^2+4(\text{triangles})=A$

$a^2+b^2+\cancel{(4\text{ triangles})}=c^2+\cancel{4(\text{triangles})}$

$a^2+b^2=c^2$

5

(2) $(a-b)^2+2ab=c^2$

$a^2-\cancel{2ab}+b^2+\cancel{2ab}=c^2$

$a^2+b^2=c^2$

$(a-b)(a-b)$

$a^2-ab-ab+b^2$

The 2 proofs were formed by the triangle activity we did in class with the cut outs.

(3) Pathorgen triples: $(2mn)^2+(m^2-n^2)^2=(m^2+n^2)^2$

$2m^2n^2=2m^2n^2$

$a=2mn$

$b=m^2-n^2$

$c=m^2+n^2$

assume that $\sqrt[7]{3}$ is rational

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational. $3^{\frac{1}{7}}$

$(3^{\frac{1}{7}})^7=\left(\frac{m}{n}\right)^7$

$3=\frac{m^7}{n^7}$

$3n^7=m^7$

$m=7^i a \quad n=7^j b$

$(7^i a)^7=3(7^j b)^7$

$7^{7i}a^7=7^{7j+1}b^7$

$7i\neq 7j+1$

contridiction

Why? explain

Hence $\sqrt[7]{3}$ is irrational

9

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

$z\to z^2+c$ set of $f(c), f(f(c)),\ldots$

This $\underline{z^2+c}$ is used to check if a certain c is in the set. If it converges then it is in the set. If it diverges then it is not in the set.

5

(b) (5 points) Define the Feigenbaum constant. Explain everything!

$\lim_{n\to\infty}\frac{r_n-r_{n-1}}{r_{n+1}-r_n}=4.669\ldots$ define r_n

2

4. (10 pts. total)

(a) (6 pts.) For any position P in the $(n^2\text{-}1)$-puzzle, let $[i, j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1, 2, 3, \ldots, n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + \mathit{inv}\,(\pi(P)) \quad ,$$

where $\mathit{inv}\,(\pi)$ is the number of inversions of the permutation π.
Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd**.
Note: You may use the lemma that if you exchange *any* **two** elements in a permutation, the number of inversions *always* changes by an odd integer.

Q

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 9 & 8 \end{pmatrix} P$$

$S = in(\pi(P)) + i + j$

$S = 0 + 3 + 3$

$S = 6$

$S = 1 + 3 + 2$

$S = 6$

Here by the lemma we change 2 elements in P. we swapped the blank (9) by 8. So the # of inversions changed by an odd integer (in this case 1).

not clear

3

(b) (4 pts)
Let

$1+4=5=inv$ $\qquad$ $3+1+3=7=inv$

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & 9 & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & 9 \\ 6 & 7 & 8 \end{pmatrix}$$

4 2 1 3 5 9 6 7 8

(3) (1) (0) (0) (0) (3) (0) (0) (0) = 7

Can you reach position Q from position P, by a sequence of legal moves? Explain!

P: $S = inv(\pi) + i + j$

$S = 5 + 2 + 2$

$S = 9 \leftarrow$ odd

Q: $S = inv(\pi) + i + j$

$S = 7 + 2 + 3$

$S = 12 \leftarrow$ even

One is even & other is odd, so it is not possible. You cannot reach.

4

5. (10 points)
Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

This answer is on the first page.

3

6. (10 points) What is the name of the following famous equation-pair?

First found by Euler

or, in fuller notation ?. $\begin{cases} u_x = v_y \quad , \quad u_y = -v_x \quad , \\ \dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y} \quad , \quad \dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x} \quad . \end{cases}$

Laplace's equation : $\dfrac{\partial^2 V}{\partial x^2} + \dfrac{\partial^2 V}{\partial y^2} + \dfrac{\partial^2 V}{\partial z^2} = 0$ X 0

What is special about the function $u(x,y) + iv(x,y)$ where $u(x,y), v(x,y)$ satisfy the above system of two equations?

Its special becaux it can be used to solve for general cubic equation ? X 0

7. (10 points) Who discovered the quaternions? What city did that person live in?

The person that discovered quaternions was Hamilton & He lived in Dublin (William Rowan Hamilton) 10

8. (10 points) What is Heron's formula, what century did Heron live in?

- $\sqrt{s(s-a)(s-b)(s-c)}$ ← Heron's formula ← Area of triangle A
- ~~Lived in B.C.~~ 18th century X 7

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

- Teacher : Issac Barrow ?
- Studied in Cambridge
- position after leaving Cambridge : Ph.D & professor X
- unusual action teacher did :

5

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

- Born in Leipzig
- Spent most life in Germany Hanover
- King that was employer of Leibnitz : King George I ?

7

11. (10 points total)

(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$$\frac{\pi}{2} = \frac{2 \cdot 2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8 \cdot 8 \cdots\cdots}{1 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7 \cdot 9 \cdot 9 \cdots\cdots}$$

this is Wallis's! X 2

(b) (5 points) State the names of two people who initiated the use of logarithms

- Euler claimed that $\log(-1) = 0$ in a letter to D'Alembert in 1747.
- D'Alembert
- Also Jacob used logarthmetic spirals.

X 0

NAME: (print!) Karine Yamouti

E-Mail address: KarineYamout@gmail.com

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTE-BOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

---

1. 5 (out of 10)
2. 5 (out of 10)
3. 7 (out of 10)
4. 4 (out of 10)
5. 5 (out of 10)
6. 0 (out of 10)
7. 10 (out of 10)
8. 6 (out of 10)
9. 4 (out of 10)
10. 3 (out of 10)
11. 10 (out of 10)

total: (out of 110)

59 · 1,134 = 67

(5) $H = \{h_1, h_2, \cdots h_m\}$ $|G| = n$, $|H| = m$

g_1 = identity

If $H \neq G$, $\exists g \in G$ s.t $g_2 \notin H$

$g_2H = \{g_2h, g_2h_2, \cdots, g_2h_m)$

$g_2h_i = g_2h_j \Rightarrow (g_2^{-1}g_2)h_i = (g_2^{-1}g_2)h_j$

$h_i = h_j$ ✗. proved by contradiction

$g_2h_i = h_j$

$(g_2h_i)(h_i^{-1}) = h_jh_i^{-1}$

$g_2(h_ih_j^{-1}) = h_jh_i^{-1}$

$g_2(e) = h_jh_j^{-1} \in H$ ✗.

$g_3 \notin H$ $g_3 \in g_2H$

$g_3H = \{g_3h_1, g_3h_2, \cdots g_3h_m\}$

$g_2h_1 = g_3h_i$

$g_3 = g_2h_ih_j^{-1}$

So $g_3 \in g_2h$ ✗.

Keep doing it until you finish (n times)

not clear

1. (10 pts.) Give **two** proofs of the Pythagorean theorem.

$(a-b)(a-b) = a^2 - ab - ab + b^2$

(1) $c^2 = (a-b)^2 + 2ab$

$c^2 = a^2 - \cancel{2ab} + b^2 + \cancel{2ab}$?

$c^2 = a^2 + b^2$

$a^2 + b^2 = c^2$

5

(2) Triangle 1 : $a^2 + b^2 + 4$(triangles)

Triangle 2 $c^2 = 4$(triangles)

$a^2 + b^2 + 4$(triangles) $= 4$(triangles)

$a^2 + b^2 = c^2$.

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

Assume by contradiction $\sqrt[7]{3}$ is rational

so, $\sqrt[7]{3} = \frac{m}{n}$

so, $3 = \frac{m^7}{n^7}$

$3n^7 = m^7$

$3(3^i n)^7 = (3^j m)^7$

$3^{7i+1-7}n = 3^{7j} m^{-7}$

$7i + 1 = 7j$

$7(j-1) = 1$

proof by contradiction.

not clear!

5

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

Mandelbrot set $Z_{n+1} = Z_n^2 + c$, $Z_0 = 0$, if sequence does not diverge part of set.

5

(b) (5 points) Define the Feigenbaum constant. Explain everything!

$$\lim_{n\to\infty} \frac{r_n - r_{n-1}}{r_{n+1} - r_n} = 4.669.$$

define r_n!

2

4. (10 pts. total)
(a) (6 pts.) For any position P in the (n^2-1)-puzzle, let $[i, j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1, 2, 3, \ldots, n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + \ inv\,(\pi(P)) \quad ,$$

where $inv\,(\pi)$ is the number of inversions of the permutation π.
Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd**.
Note: You may use the lemma that if you exchange *any* **two** elements in a permutation, the number of inversions *always* changes by an odd integer.

If P is a position in the (n^2-1) sliding puzzle and $\pi(P)$ is corresponding permutation where the blank is replaced by n^2: the location of the blank is $[i,j]$

0

$S(P) = inv(P) + i + j$

If position Q is reachable from position P by a finite number of legal moves, then $S(P) = S(Q)$ have the same parity.

← You just copied the theorem! You had to prove it!

(b) (4 pts)
Let

$inv(P) = 1+4 = 5$

$$S_1 = P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & 9 & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad S_2 = P' \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & 9 \\ 6 & 7 & 8 \end{pmatrix}$$

$inv(P') = 3 + 1 + 3 = 7$

Can you reach position P' from position P, by a sequence of legal moves? Explain!

$S_1 = inv(\pi) + i + j$ $\qquad$ $S_2 = inv(\pi) + i + j$

$S_1 = 5 + 2 + 2$ $\qquad$ $S_2 = 7 + 2 + 3$

$S_1 = 9$, odd. $\qquad$ $S_2 = 12$, even.

4

You cannot reach position P' from position P by a sequence of legal moves.

5. (10 points)
Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

see front page

5

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x, y) + iv(x, y)$ where $u(x, y), v(x, y)$ satisfy the above system of two equations?

Dirichlet principle ?

0

7. (10 points) Who discovered the quaternions? What city did that person live in?

Hamilton found the quaternions. He lived in Dublin, Ireland.

10

8. (10 points) What is Heron's formula, what century did Heron live in?

The "Heronic" formula for the area of a triangle

$A = \sqrt{s(s-a)(s-b)(s-c)}$, roughly 11th century

6

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

Isaac Barrow was his teacher

sorry, I have so much infor on Newton But not what you asked

4

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

He ~~lived~~ was born in Leipzig

3

11. (10 points total)

(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$\frac{2}{\pi} = \cos\frac{\pi}{4} \cos\frac{\pi}{8} \cos\frac{\pi}{16} \cos\frac{\pi}{32}$

5

(b) (5 points) State the names of two people who initiated the use of logarithms

Napier : Briggs.

5

/

NAME: (print!) Lauren McKay

E-Mail address: lauren.mckay@rutgers.edu

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

---

1. 5 (out of 10)

2. 10 (out of 10)

3. 3 (out of 10)

4. 6 (out of 10)

5. 2 (out of 10)

6. 0 (out of 10)

7. 10 (out of 10)

8. 8 (out of 10)

9. 4 (out of 10)

10. 4 (out of 10)

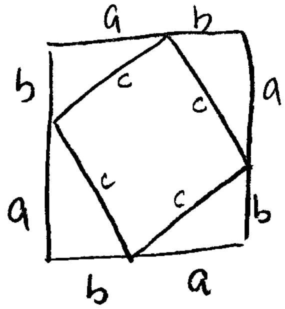
11. 6 (out of 10)

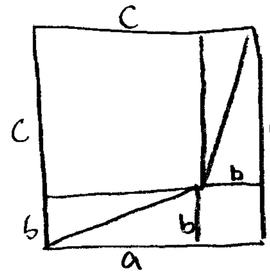
---

total: (out of 110)

$58 \cdot 1.134 = 66$

1. (10 pts.) Give two proofs of the Pythagorean theorem.





5

triangles of a△b will fit in both

2nd ~~proof~~ proof?

$a^2 + b^2 + 4\cancel{(\tfrac{1}{2}ab)} = c^2 + 4\cancel{(\tfrac{1}{2}ab)}$
Δ's Δ's

leaving us with

$a^2 + b^2 = c^2$

and therefore cancel out because they appear on both sides

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

Suppose it is rational

$\sqrt[7]{3} = \frac{m}{n} \rightarrow 3 = \frac{m^7}{n^7} \rightarrow 3n^7 = m^7$

$m = 3^i a$ $i>0$ $(\gcd(3,a)=1)$

$n = 3^j b$ $j>0$ $(\gcd(3,b)=1)$

plugging in

$3(3^j b)^7 = (3^i a)^7$

$3^{7j+1} b^7 = 3^{7i} a^7$

seeing both sides are by 3 but $3^{7j+1} \neq 3^{7i}$

$7j+1 \neq 7i$ didn't parities again making it disjoint and irrational

10

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

$Z = Z^2 + c$ where $z_0 = 0$

1

(b) (5 points) Define the Feigenbaum constant. Explain everything! chaos!

$\lim_{n\to\infty} \frac{r_n - r_{n+1}}{r_{n+1} - r_n} = 4.669\ldots$ with period doubling

what is r_n?

2

4. (10 pts. total)

(a) (6 pts.) For any position P in the (n^2-1)-puzzle, let $[i, j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1, 2, 3, \ldots, n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + \text{ inv}\,(\pi(P)) \quad ,$$

$inv(\pi(P))$ # of times a bigger # is in front of smaller #.

where $inv\,(\pi)$ is the number of inversions of the permutation π.

Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd**.

Note: You may use the lemma that if you exchange *any* **two** elements in a permutation, the number of inversions *always* changes by an odd integer.

lemma: exchanging any 2 elements at position (i,j) with inversion π_{ij} then $inv(\pi) - inv(\pi_{ij}) =$ odd#!

Why? In order for a permutation to be reachable it must be of the same parity. If they are opposite then it would be impossible.

not clear

2

$S(P) = i_1 + j_1 + (\text{odd\#} + inv\,\pi_{ij})$ $S(Q) = i_2 + j_2 + (inv\,\pi - \text{odd\#})$

(b) (4 pts)

Let

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & \\ 6 & 7 & 8 \end{pmatrix} \quad .$$

Can you reach position ~~P~~ Q from position P, by a sequence of legal moves? Explain!

$S(P) =$ 1 2 3 5 9 4 6 7 8 (1) (4) $inv(P) = 5$, $i = 2$ $j = 2$ $S(P) = 5 + 2 + 2 = 9$

$S(Q) =$ 4 2 1 3 5 9 6 7 8 (3) (1) (3) $inv(Q) = 7$, $i = 2$ $j = 3$ $S(Q) = 7 + 2 + 3 = 12$

4

Parity of $S(P)$ & $S(Q)$ are different therefore unreachable.

5. (10 points)

Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

$|H| = m$ $\{h_1, \ldots, h_m\}$

$|G| = n$

2

Lemma: $a \notin H$ with no overlap with elements in H

then $aH = \{ah_1, ah_2, \ldots, ah_m\}$

$ah_i = ah_j$

$a^{-1}(ah_i) = a^{-1}(ah_j)$

$(a^{-1}a)h_i = (a^{-1}a)h_j \rightarrow h_i = h_j$ contradiction cannot be true

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x, y) + iv(x, y)$ where $u(x, y), v(x, y)$ satisfy the above system of two equations?

Canonical form relation between dynamics & Hamilton-Jacobi diff. equation & contact transformations

0

7. (10 points) Who discovered the quaternions? What city did that person live in?

William Hamilton lived in Dublin, Ireland

10

8. (10 points) What is Heron's formula, what century did Heron live in?

$A = \sqrt{s(s-a)(s-b)(s-c)}$ ~ 200 BC? don't know what century you'd call this

1st century?

8

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

University of Cambridge, Fermat

4

position → Royal A

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

Leipzig, Germany, in ~~France~~ (Europe) generally, William III

4

11. (10 points total)
(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$$\frac{\pi}{2} = \frac{2 \cdot 2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8 \cdot 8 \cdots}{1 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7 \cdot 9 \cdots}$$

← this is Wallis

(b) (5 points) State the names of two people who initiated the use of logarithms

6

Briggs & Napier

NAME: (print!) Weikun Lu

E-Mail address: weikun.lu@rutgers.edu

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

---

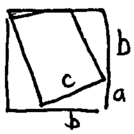
1. 5 (out of 10)
2. 10 (out of 10)
3. 9 (out of 10)
4. 3 (out of 10)
5. 0 (out of 10)
6. 0 (out of 10)
7. 10 (out of 10)
8. 0 (out of 10)
9. 5 (out of 10)
10. 5 (out of 10)
11. 7 (out of 10)

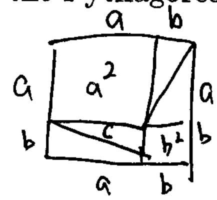
---

total: 52 (out of 110)

1.134 = 59

1. (10 pts.) Give **two** proofs of the Pythagorean theorem.





$(a+b)^2 = a^2+b^2+2ab$

area: $c^2+\frac{ab}{2}\cdot 4$

$a^2+b^2+2ab = c^2+2ab$

$\therefore a^2+b^2=c^2$

$(a+b)^2 = a^2+2ab+b^2$

$2ab + c^2 = a^2+b^2+2ab$

$c^2=a^2+b^2$

5

2nd proof?: X

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

suppose $\sqrt[7]{3}$ is rational

then $\sqrt[7]{3}=\frac{m}{n}$ m, n all positive integers

$3=\frac{m^7}{n^7}$

$3n^7=m^7$

$m=3^i\cdot\bar m$ $\quad i\ge 0$

$n=3^j\cdot\bar n$ $\quad \bar m,\bar n$ odd

$3(3^j\cdot\bar n)^7=(3^i\cdot\bar m)^7$

$3\cdot 3^{7j}\cdot\bar n^7 = 3^{7i}\cdot\bar m^7$

$3^{7j+1}\cdot\bar n^7=3^{7i}\cdot\bar m^7$

$7j+1=7i$

$7(i-j)=1$

no way

so contradiction.

10

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

~~the set of complex numbers~~

$f_c(z)=f(z)+c$

the sequence is $f_c(z)$, $f(f_c(z))$, $f(f(f_c(z)))$

it will have a a boundary in the absolute value

5

(b) (5 points) Define the Feigenbaum constant. Explain everything!

$\lim_{n\to\infty}\frac{r_n-r_{n-1}}{r_{n+1}-r_n}=4.669\ldots$

what is r_n?

2

4. (10 pts. total)
(a) (6 pts.) For any position P in the (n^2-1)-puzzle, let $[i, j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1, 2, 3, \ldots, n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + \ inv\,(\pi(P)) \quad ,$$

where $inv\,(\pi)$ is the number of inversions of the permutation π.
Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd**.
Note: You may use the lemma that if you exchange *any* **two** elements in a permutation, the number of inversions *always* changes by an odd integer.

Lemma: $inv(\pi(ij)) - inv(\pi) =$ odd when exchange two elements.

~~so when they are both even~~

because when exhange two elements, the inversion will change, $i+j$ change by 1, then in order to keep the $inv(\pi_{ij}) - inv(\pi)$ lemma hold, the P and Q must have the same parity.

3

not clear!

(b) (4 pts)
Let

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & \\ 6 & 7 & 8 \end{pmatrix} \quad .$$

0

Can you reach position Q from position P, by a sequence of legal moves? Explain!

$inv(P) = 0+0+0+\cancel{0}+4+0+0+0+0 = \cancel{4}\,5$

$inv(Q) = 3+1+0+0+0+3 = 7$

$inv(Q) - inv(P) = 7-4 = 3 =$ odd

so reachable.

What are $S(P), S(Q)$?

5. (10 points)
Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

$|H|, |G|$

0

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad , .$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x,y) + iv(x,y)$ where $u(x,y), v(x,y)$ satisfy the above system of two equations?

Newton's equation.

0

7. (10 points) Who discovered the quaternions? What city did that person live in?

William Rowan Hamilton

Dublin

10

8. (10 points) What is Heron's formula, what century did Heron live in?

~~Axiom of triangle~~

Axiom of triangle

first century

0

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

Cambridge. Isaac Barrow,

he tried to make his position by "theory of prime and ultimate ratios"

5

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

Leipzig, Germany.

King 14

5

11. (10 points total)

(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$\frac{2}{\pi} = \cos\frac{\pi}{4} \cos\frac{\pi}{8} \cos\frac{\pi}{16} \cdots$

7

(b) (5 points) State the names of two people who initiated the use of logarithms

Steven, John Neper

/

NAME: (print!) ZongJie Deng

E-Mail address: zjd8@scarletmail.rutgers.edu.

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

---

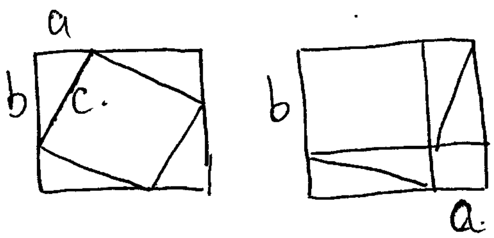
1. 10 (out of 10)
2. 10 (out of 10)
3. 6 (out of 10)
4. 8 (out of 10)
5. 10 (out of 10)
6. 3 (out of 10)
7. 0 (out of 10)
8. 0 (out of 10)
9. 0 (out of 10)
10. 0 (out of 10)
11. 5 (out of 10)

---

total: (out of 110)

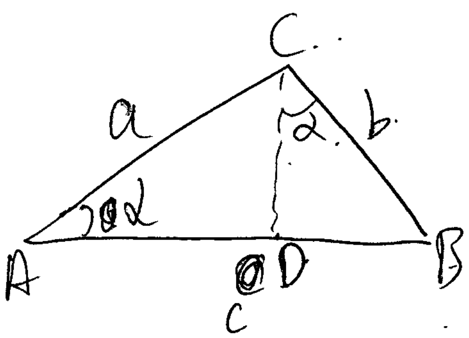
$52 \cdot 1.134 = 59$

1. (10 pts.) Give **two** proofs of the Pythagorean theorem.



$c^2 + 4\times\frac{1}{2}ab = a^2+b^2+4\times\frac{1}{2}ab$

$\Rightarrow c^2 = a^2+b^2.$



$S_\Delta = kc^2 = ka^2 + kb^2$, Since $\Delta ABC \sim \Delta ACD \sim \Delta CBD$.

$\Rightarrow c^2 = a^2+b^2$,

10

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

Assume $\sqrt[7]{3}$ is rational;

$\exists\, m,n \in \mathbb{N}$: $\sqrt[7]{3} = \frac{m}{n}$, $\gcd(m,n)=1$.

$\Rightarrow 3n^7 = m^7$.

$\Rightarrow 3 \mid m^7$

$\Rightarrow 3 \mid m$, Since 3 is prime number

$\Rightarrow 9 \mid m^7$.

$\Rightarrow 9 \mid 3n^7$.

$\Rightarrow 3 \mid n^7$

$\Rightarrow 3 \mid n$, Since 3 is prime number

$\Rightarrow \gcd(m,n) > 1$, ※. $\therefore \sqrt[7]{3}$ is irrational.

Q.E.D

10

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

~~let $z_n = z_{n-1}$ $z_n = z_{n-1}+c$~~

Mandelbrot set $= \{c \in \mathbb{C} \mid z_n$ diverges, where $z_n = z_{n-1}+c\}$

5

(b) (5 points) Define the Feigenbaum constant. Explain everything!

1

$\lim_{n\to\infty} \frac{r_{n-1}-r_{n-2}}{r_n-r_{n-1}}$

what is r_n?

4. (10 pts. total)

(a) (6 pts.) For any position P in the $(n^2$-1)-puzzle, let $[i,j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1,2,3,\ldots,n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + \; inv\,(\pi(P)) \quad ,$$

where $inv\,(\pi)$ is the number of inversions of the permutation π.

Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd.**

Note: You may use the lemma that if you exchange *any* **two** elements in a permutation, the number of inversions *always* changes by an odd integer.

$S(P) = i+j+inv(\pi(P))$,

$S(Q) = i+j(\pm)1 + \cancel{inv(\pi(P))}\, in(\pi(Q))$.

$= i+j(\pm)1 + in(\pi(P)) + 2k-1$, for some $k \in \mathbb{N}$.

$= i+j+ in(\pi(P)) + 2m$, for some $m \in \mathbb{N}$

$\therefore\ S(Q) \equiv S(P) \bmod 2$.

6

(b) (4 pts)

Let

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & \\ 6 & 7 & 8 \end{pmatrix} \quad .$$

Can you reach position P' from position P, by a sequence of legal moves? Explain!

$S(P) = 2+2+5 = 9$

$S(Q) = 2+3+\cancel{10} = 15$

7

$\Rightarrow S(P) \equiv S(Q) \bmod 2$,

P' can be reached from P by a sequence of legal moves.

2

5. (10 points)

Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

10

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x, y) + iv(x, y)$ where $u(x, y), v(x, y)$ satisfy the above system of two equations?

Laplace equation, the function is analytic on the whole complex plane. .

3

7. (10 points) Who discovered the quaternions? What city did that person live in?

0

8. (10 points) What is Heron's formula, what century did Heron live in?

0

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

0

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

0

11. (10 points total)
(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$\cancel{\frac{2}{\pi} = \cos 1 \cos 2 \cos 3 \cdots \cos n \ldots}$

$\frac{2}{\pi} = \cos\frac{\pi}{1} \cos\frac{\pi}{2} \cos\frac{\pi}{3} \cdots \cos\frac{\pi}{n} \cdots$

5

(b) (5 points) State the names of two people who initiated the use of logarithms

Euler and d'Alembert.