Solutions to Real Quiz # 4 for Dr. Z.'s MathHistory

1. (2 points) What is the difference between *arithmetica* and *logistics* in Greek mathematics?

Ans. *arithmetica* was *pure mathematics*, and *logistics* was applied, practical, engineering-oriented mathematics.

2. (1 points) Did the ancient Greeks have a notion of real numbers? Was the Pythagorean theorem a relation between the lengths of the sides of a right-angled triangle?

Ans.: No! They were confused by lengths (because they were usually *irrational*). The Pythagorean theorem was a relation between the *areas* of the sides of a right-angled triangle. So they *never* wrote it as $c = \sqrt{a^2 + b^2}$, but left it as $c^2 = a^2 + b^2$.

3. (2 point) What is the best known achievement of Hindu mathematics?

Ans. The positional decimal system (using the most important digit, zero!).

4. (2 points) Prove that there are infinitely triples of **positive** integers a, b, c such that

$$a^2 + b^2 = c^2 \quad .$$

Sol. of 4

I claim that for any **positive** integers m, n, with $m > n > 0$ the three integers a, b, c given by the formulas

$$a = m^2 - n^2 \quad , \quad b = 2mn \quad , \quad c = m^2 + n^2$$

always satisfy $a^2 + b^2 = c^2$. Obviously there are infinitely many choices! (in fact, *doubly* infinite choices).

We have to prove $c^2 - a^2 - b^2 = 0$. We will use *high-school algebra.*

$$c^2-a^2-b^2 = (m^2+n^2)^2-(m^2-n^2)^2-(2mn)^2 = (m^2)^2+2\cdot m^2\cdot n^2+(n^2)^2-((m^2)^2-2\cdot m^2\cdot n^2+(n^2)^2)-4m^2n^2$$

$$m^4+2m^2n^2+n^4-(m^4-2m^2n^2+n^4)-4m^2n^2 = m^4+2m^2n^2+n^4-m^4+2m^2n^2-n^4-4m^2n^2 = 0 \quad ,$$

QED.

Comment: Some people answered correctly that $(3n, 4n, 5n)$ is such an infinite family. I gave them full credit, even though these are 'boring' consequenes of the fact that $(3, 4, 5)$ is a Pythagorean triple. I should have said 'infinitely, not trivially-equivalent' Pythagorean triples, but I did not, so tough on me.

5. (3 points) There are two people A and B each of them is either a Truth-Teller or a Lie-Teller. A Truth-Teller *always* tell the truth. A Lie-Teller *always* tell lies.

Which of the possible scenarios (if any!)

(a) Both A and B are truth-tellers (b) Both A and B are Lie-Tellers (c) A is a Truth-Teller and B is a Lie-Teller (d) A is a Lie-Teller and B is a Truth-Teller

are compatible with the following statements uttered by A and B

A: B is a Lie-Teller

B: A is a Lie-Teller

EXPLAIN!

Sol. of 5: If A is a Truth-Teller then B is a Lie-Teller, and when B utteres 'A is a Lie-Teller' it is a lie, consistent with the fact that B is a Lie-Teller.

So **one possible** scenario is

A is a Truth-Teller, and B is a Lie-Teller.

If A is a Lie-Teller, then the statement: 'B is a Lie-Teller' is a lie, hence B is a Truth-Teller. When B says: 'A is a Lie-Teller', it is a true statement, consistent with the fact that B is a Truth-Teller.

Hence **another possible** scenario is

A is a Lie-Teller, and B is a Truth-Teller.

That's the only possibilities.

Ans. to 5: The only possible scenarios are (c) and (d).