

Solutions to the Real Quiz # 1 for Dr. Z.'s MathHistory

1. (1 points) What are the names of the two papyri that gave us the most information about the math of ancient Egypt?

Ans. to 1: The Rhind papyrus and the Moscow papyrus.

2. (2 points) What was the Egyptian formula for the area of a circle with diameter d ? What was the implied value of π ?

Ans to 2: If d is the diameter, then the area is $(d - \frac{d}{9})^2$. The implied value of π is $(\frac{16}{9})^2 = \frac{256}{81} = 3.16\dots$

3. (2 points) What was the Egyptian formula for the volume of the frustum of a square pyramid for which a and b are the lengths of the sides of the squares, and h is the height?

Ans. to 3:

$$\frac{h}{3}(a^2 + ab + b^2) \quad .$$

4. (2 points) Convert one thousand and one (base ten) to base thirteen. Use the following abbreviations for the digits that represent our ten, eleven, and twelve

ten=A , eleven=B, twelve=C

Sol. to 4: The largest power of 13 smaller than 1001 is $13^2 = 169$. Dividing by 13 gives

$$1001 = 77 \cdot 13 + 0$$

(the remainder is 0.) The largest power of 13 smaller than 77 is $13^1 = 13$. Dividing by 13 gives

$$77 = 5 \cdot 13 + 12$$

Going back above, we have

$$1001 = (5 \cdot 13 + 12) \cdot 13 + 0 = 5 \cdot 13^2 + 12 \cdot 13^1 + 0 \cdot 13^0 \quad .$$

But 12 is called C , so we get

Ans. to 4 : $5C0_{13}$.

5. (3 points) Find the two smallest positive integers n , that have the property that

- If you divide n by 6 you get remainder 5 .
- If you divide n by 7 you get remainder 6 .

Sol. to 5:The ‘stupid’ way is to compile a table of the mapping

$$x \rightarrow (x \pmod{6}, x \pmod{7}) \quad ,$$

for x from 0 to 41. This is correct, but very time consuming, and the answer is $n = 41$.

A better way is to note that $n \pmod{6} = -1$, and $n \pmod{7} = -1$. By luck they are the same, so $n = -1$ is an obvious answer. Alas, it is not positive. But if you add to it $6 \cdot 7 = 42$, you get that the smallest positive answer is $n = -1 + 42 = 41$.

To get the second-most positive answer, you add 42 one more time, getting $41 + 42 = 83$.

Ans. to 5: The smallest positive answer is $n = 41$, and the second-smallest is $n = 83$.