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History of Math
1 May 2017

The History and Evolution of the Hindu-Arabic Numerals

Ever since I started studying mathematics, I fell in love with the discipline. Mathematics is a logical field of study, and everything can be proven objectively. This completeness of everything making sense is what most appealed to me about mathematics, that and the symbols used in it. The subject of mathematics is not like any other in learned in school because of its unique use of symbols to convey and expound upon the ideas presented in it. Certainly the alphabet which is used to write in English class or the diagrams drawn in a science class have their own innate charm, but there is nothing like the flexibility and deep meaning of the symbols used in mathematics. These symbols are truly fascinating when examined carefully, which is not something a student – or anyone, for that matter – often does. The most precious of the symbols used in mathematics are the numerals themselves. Without these Arabic numerals from "0" to "9," the mathematics we know today would not exist. One benefit of studying mathematics is that it organizes one's thought processes and teaches its student to make logical connections. In that spirit, it is similarly instructive to look back on the semiotic history of these ten symbols to investigate their origins and how they evolved and developed throughout time to the numerals we use today. To start, one must search for the rudimentary roots, the place of origin, of the ten numerals.

Given that the numbers "0," "1," "2," "3," "4," "5," "6," "7," "8," and "9" are called the "Arabic numerals," one would assume that these numbers took shape somewhere in Arabia. This, however, is not at all true. In fact, these numerals evolved from numerals used in India, and the Arabs, who played a major role in propagating these forms, clearly state that India is from where these numerals originated (Smith and Karpinski 4-6). Of course, the present forms are not what first appeared or were first devised in India centuries ago nor was the decimal positional, or place-value, system in place at the inception of

the predecessors to our numbers. In the beginning, there were many different systems of numbers used in India. The three principal groups were the Kharoṣṭhī numerals, the Brāhmī numerals, and the alphabetical "word and letter forms" of the numbers (Smith and Karpinski 4-6). It is this second system which appear to be the prototypes of our so-called "Arabic numerals," or rather our "Hindu-Arabic numerals" ("Indian numerals," Smith and Karpinski 22). These early Brāhmī numerals lacked the current place-value system due to there being no symbol for zero. In the first or second century A.D., the numerals took the following forms: one was written as a horizontal stroke, two was written as two horizontal strokes, three was written as three horizontal strokes, four was written as a cross, five resembled our "h," six was halfway between our current "6" and the Greek "ϕ," seven was written as a more rounded version of our "7," eight was written as a straight-lined "s" rotated ninety degrees to the left, and nine resembled the top portion of our "?" ("Indian numerals," Smith and Karpinski 24). Very many theories, some more absurd than others, attempt to support claims of the origins of these numerals, especially the first three, stating, for example, that they were brought over by the Chinese, who got it from migrations from Sumer whose civilization used horizontal strokes despite the subsequent Babylonians using vertical ones; Smith and Karpinski, however, conclude that these numerals were indeed indigenous to India (28-35). Naturally, the lack of a positional system led to the Brāhmī numeric system to consist of different symbols for various multiples of ten, until the decimal system took shape, adopting the first nine Brāhmī symbols and annexing the symbol for zero, which does not happen until centuries later (Smith and Karpinski 24).

As a culture, the Western peoples and all others who employ the Hindu-Arabic numerals are much indebted to the Hindu peoples who, not only were responsible for the nine quantifiable digits we use today, but also introduced the concept of base-ten place value or positionality and a symbol for nothingness, our modern-day zero. First and foremost, as these new notions developed, so too did the Brāhmī numerals. By the early fourth century A. D., a new numerical system had evolved from them,

called the Gupta numerals, whose symbols for four and eight look much more like our current symbols ("Indian numerals"). It is in these later Hindu forms, circa the sixth century A.D., that a positional system emerges in the form of words; for example, the number written as "8,443,682,155" would have been read as "8 padmas, 4 vyarbudas, 4 koṭis, 3 prayutas, 6 lakṣas, 8 ayutas, 2 sahasra, 1 śara, 5 daśan, 5" (Smith and Karpinski 42). Unlike other languages at this time – and even contrasting with the way the number would currently be read in English –, each digit of the number has its place value designated by a distinct word. The oldest document utilizing this decimal positional system that has been dated is a legal one in the Chhedi calendar year of 346 or, roughly, 594 A.D., whereas the first undoubted inscription using the base-ten place-value system is one found at Gwalior with the Vikrama calendar year of 933 or about 876 A.D. ("Indian numerals). Similar to the theories behind the Brāhmī numerals, O'Connor and Robertson enumerate a number of theories behind the positional system devised in India; one says that the Indians were inspired by the sexagesimal, or base-sixty, positional system of the Babylonians transmitted to them by the Greeks, another that the semi-positional Chinese numeric system served as an influence, and a third that the place-value system was again native to India itself ("Indian numerals"). Of course, the decimal positional system could not have possibly flourished to the full extent to which it has without the invention of the symbol for zero, which was, for all intents and purposes, the missing piece of the puzzle for a decimal numeric place-value system.

This missing piece of the jigsaw puzzle arrived as the Gupta decimal positional number system developed. The ancestor of what the present-day students of mathematics know as zero got its start in about 500 A.D. Although the Babylonians had used a similar symbol used in fractions, it was never used in calculations, nor did it appear in any numbers that were multiples of sixty, as theirs was a sexagesimal number system; the Greeks also used their lowercase omicron as a symbol for "oudén" or "vacant" (Smith and Karpinski 51-52). This tenth symbol also called India home, and its name was śūnyabindu, although it is commonly shortened to simply śūnya, which signifies "the void" (Smith and Karpinski 43).

Āryabhaṭa, who worked with extracting roots at this time but used a different numerical system, often discusses “the void” in his work, but it is unclear whether he was referring to an actual symbol for the concept like the modern zero or if he was just referring to general nothingness (Smith and Karpinski 43-44). In the same century, Varāha-Mihira also used the word śūnya when discussing numerals, suggesting that it was conceived as a symbol proper and lending credence to the possibility of Āryabhaṭa having done the same earlier (Smith and Karpinski 44). There does, however, seem to be some conflicting evidence, so it is entirely possible, as Smith and Karpinski point out, that only the elite Hindu mathematicians were cognizant of this new zero symbol, while the common people and the merchants/traders did not yet fully understand it (45). The first undisputed occurrence of the zero in India is an inscription in Gwalior circa 876 A.D., represented as a circle, whereas the disputed Bakhṣālī Manuscript uses a dot for zero, so it probably came before this inscription (Smith and Karpinski 52). The new numeral spread all throughout India, except for the southern part of the subcontinent where, curiously, the Tamil and Malayalam peoples still prefer to adhere to their ancient numeric systems (Smith and Karpinski 52). The arrival of this new mathematical digit brought with it the need to define its characteristics.

Other than on displays such as inscriptions, the novel zero symbol was utilized in arithmetic as well, and it had different forms throughout these early times of its history. In the early seventh century A.D., Brahmagupta, without a symbol for zero, discusses its arithmetical properties; later, another Indian scientist, Bhāskara, discusses the characteristics of zero more rigorously, although he makes the commonly-known error of dividing by zero (Smith and Karpinski 52). Circa 830 A.D., the book Ganita-Sāra-Saṅgraha by the mathematician Mahāvīrācārya circa 830 A.D. also discusses the new number but without the positional system (Smith and Karpinski 53). As mentioned before, the first zero symbol was a dot and then a cross; the Arabs actually adopted this punctual depiction of zero and still use to this day in the Eastern Arabic numeral system as well as in the Persian numeral system (Smith and Karpinski 53).

In fact, the Arabs adopted, in al-Biruni's words, "the most regular figures" of the Indian numerals, which, from the seventh century, started to evolve from the Gupta numerals to the Nāgarī numerals, the shortened name for the Devanāgarī numerals ("Indian numerals"). These numerals, unlike the Gupta or the Brāhmī from before, finally include this zero symbol, but in the form of a circle, probably influenced from the old Brāhmī symbol for the number ten, which resembles a circle with two spikes coming out from it on the right side (Smith and Karpinski 54). The name of zero also evolved along with its shapes and forms: from the Sanskrit "śūnya" came the Arabic "aṣ-ṣifr," which led to Leonard of Pisa [Fibonacci] calling it "zephirum" in the year 1202 and Maximus Planudes calling it "tziphra" in the year of 1330 (Smith and Karpinski 57). From the Arabic come the English word "cipher" and the French word "chiffre," although, in both languages, the meanings have been altered altogether to mean any numerical figure in general (Smith and Karpinski 58). Other derivations of the Arabic name include "zephyr" and the present-day "zero," and some alternate denominations that were also utilized for the number include "sipos," "wheel," "circulus," "das Ringlein" in German, "circular note," "theca" from the Greek letter theta which it resembled, "omicron," and "null" and its derivatives "nihil," "nulla," and the French term "rien" (Smith and Karpinski 60-62). It is this new digit, the subsequent nine Nāgarī digits, and the decimal positional system that formed the basis of the Arabic numerals used throughout Europe and around the world today.

What may strike the reader as interesting and perhaps a bit mysterious is how the numerals in India "traveled" all the way to Europe. Smith and Karpinski make a case for the copious amount of trading among the merchants of different lands before all of these Indian developments of a decimal place-value system and of zero; they would travel, sell their wares, and exchange money for goods, so anyone frequenting a marketplace at that time would be at least familiar with the various numerical systems of diverse regions of the world at that time. However, all of these number systems amount to nothing if they are all similar to one another; in other words, since there were no scholars studying the

numerical representations of faraway lands seen in the marketplaces at the time and since none of these systems were so unique as to have, say, a symbol for the abstract concept of nothingness, there was no number system at the time that was worthy or special enough to record (Smith and Karpinski 83, 90). Only in 773 A.D. was the Hindu system brought into Arabia, when an Indian astronomer was invited to the court of a caliph; his work was translated into Arabic by Al-Fazārī (Smith and Karpinski 92). Soon enough, a great number of Arabic scholars were working with the Indian numerals and the Indian arithmetic, making sure to credit the science and the figures as Indian (Smith and Karpinski 92). However, the Arab empire was quite extensive, so it was not easy to keep all the material being learned and disseminated "on the same page." These Arabic writers and scholars had the difficult task of translating material from one language to another, which can be rather difficult when one is all too unfamiliar with the symbols and notations used in the source language. While some writers like Al-Khowārazmī copied the numerals correctly, leading to al-ġobār [the dust] numerals, referring to the dust abacus on which these numbers were represented, other scribes did not ("The Arabic Numeral System," Smith and Karpinski 98). Arabic is written from right to left, and the scrolls, rolled out in front of them on their laps, on which the scribes would write was wound up on the left side, from right to left; therefore, they would have to write in vertical lines from top to bottom so that, when the scroll was unrolled vertically to be read later, the Arabic could be read properly from right to left ("The Arabic Numeral System"). This lack of experience with Indian numbers lead some scribes to write the Nāgarī "2" and "3," which look like the modern-day versions but with the stroke continuing downwards for both, right-side-up, which lead to them being improperly turned ninety degrees clockwise; these turned versions are the modern-day versions of the Eastern Arabic and Persian number systems, whereas al-Khowārazmī's al-ġobār numerals were brought to Spain by the Moors ("The Arabic Numeral System," Smith and Karpinski 65, 98).

With the numerals brought to the European continent, it still took some time before they began to be used. It is true that the Europeans were used to and preferred to use their own number systems, whether that be the Greek or the Roman system, for some time. The movement of travelers and pilgrims, of ambassadors and scholars, and of merchants and traders facilitated this wealth of information exchange (Smith and Karpinski 99-105). This age was marked by the famous journeys of Marco Polo circa 1271 and the not-so-famous wanderings of his father Niccolò and uncle Maffeo Polo; "there was abundant intercourse between the East and West for some centuries before the Hindu numerals appear in any manuscripts in Christian Europe" (Smith and Karpinski 107, 109-110). Slowly but surely, the al-ġobār numerals were studied by such scholars as the Frenchmen Gerbert and Raoul/Radulph of Laon near the turn of the first millennium A.D.; these numerals were given various other names throughout the years including "apices," "caracteres," "figures, ciphers, signs, elements, and characters" (Smith and Karpinski 112-113, 118-119). Finally, in 1202, came Leonardo Fibonacci's book Liber Abaci, which used these numerals in all of the business computations therein (Smith and Karpinski 131). With some time and some more dissemination of information, these developing numerals came to be known throughout Europe and are the ten digits we use today.

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