Cassandra Sanchez 4/25/17
History of Math Final Paper Spring 2017

# Anna Johnson Pell Wheeler and Her Contributions to Mathematics

Anna Johnson Pell Wheeler is a 20th century American mathematician. She is best known for her contributions to the field of integral equations, as well as for having "fostered women's participation in mathematics" (Boman, 2001, pg. 1). Johnson Pell Wheeler's accomplishments and contributions to mathematics are extensive and stretch over several decades.

## Early Life

Anna Johnson was born on May 5, 1883 to Andrew Gustav Johnson and Amelia Friberg in a town then known as Calliope, Iowa (Riddle, 2016; O'Connor & Robinson, 2014). Johnson's parents were immigrants from Lyrestad in Skaraborglänm, Sweden (O'Connor & Robinson, 2014). They arrived in the United States in 1872 and settled in as farmers in the Dakota Territory (O'Connor & Robinson, 2014). Johnson is the youngest of three surviving children (O'Connor & Robinson, 2014). Anna Johnson had a particularly close relationship with her older sister Esther Johnson (b. 1879), who also had a strong interest in mathematics (O'Connor & Robinson, 2014). While in Calliope, the Johnson family lived with their extended family (Riddle, 2016; Grinstein & Campbell, 1982, pg. 38). There are no records indicating that Johnson attended school while the family lived in Calliope (Grinstein & Campbell, 1982, pg. 38).

In 1892, the family moved to Akron, Iowa where Johnson's father tried his hand as a furniture dealer and then as an undertaker (O'Connor & Robinson, 2014). While in Akron, Johnson attended public schools (O'Connor & Robinson, 2014). Anna Johnson's mother was displeased with the deficiencies her own education so she ensured that her daughters received a quality education (O'Connor & Robinson, 2014). Additionally, Esther Johnson saw that Anna Johnson had mathematical talents and encouraged her to study further (O'Connor & Robinson, 2014). Anna Johnson graduated in 1899 from a public high school that offered an education through the eleventh grade (Riddle, 2016; Grinstein & Campbell, 1982, pg. 38).

Anna Johnson attended the University of South Dakota (Riddle, 2016). The University of South Dakota was a state-supported and coeducational institution (Hunger Parshall, 2015, pg. 77). Johnson spent her first year at the university as a "sub-freshman" to complete the entrance requirements because her high school was unable to provide her with the necessary coursework (Grinstein & Campbell, 1982, pg. 38). This left Johnson with three years to complete her degree requirements (Grinstein & Campbell, 1982, pg. 38). Alexander Pell, a mathematics professor at the university, noticed Johnson's mathematical talents and persuaded her to study mathematics and pursue a career in it (Riddle, 2016). Pell was a Russian emigrant who had fled Russia and received his PhD in mathematics in 1887 from John Hopkins University (Riddle, 2016). Johnson's sister Esther also attended the university (Riddle, 2016). In 1900, Johnson wrote to her sister expressing her desire to study mathematics in Germany (O'Connor & Robinson, 2014). Johnson and her sister rented a room from Alexander Pell and his wife (Riddle, 2016).

In her first year of college, 1889-1900, Johnson studied college algebra and trigonometry (Grinstein & Campbell, 1982, pg. 38). The following year she studied modern geometry, the theory of equations, and solid analytic geometry (Grinstein & Campbell, 1982, pg. 38). In her third year, 1901-1902, Johnson took plane analytic geometry, six credits of calculus, and six credits of analytical mechanics (Grinstein & Campbell, 1982, pg. 38). In her final year she took 18 credits of mathematics, studying the theory of substitutions and potential, partial differential equations, Fourier Series, and differential equations (Grinstein & Campbell, 1982, pg. 38). In addition, Johnson studied English, Physical Culture, History, German, Latin, French, Physics and Chemistry

(O'Connor & Robinson, 2014). Johnson received A's in every class except English, Physical Culture, and History, in which she received B's (Grinstein & Campbell, 1982, pg. 38). In addition to her coursework, Johnson served as Secretary-treasurer of the French Club (O'Connor & Robinson, 2014). Johnson's senior quote was "I know mathematics better than my own name" (Grinstein & Campbell, 1982, pg. 39). Pell encouraged her to continue studying math after she graduated (Riddle, 2016). She graduated with a bachelor's degree in mathematics in 1903 (Riddle, 2016).

**Graduate Studies**

Johnson received two master's degrees in the next two years. Johnson won a scholarship to study in the University of Iowa's graduate program in mathematics (O'Connor & Robinson, 2014; Riddle, 2016). In the year she studied at the University of Iowa, she took five courses in mathematics, one course in philosophy, and taught a "first year" calculus course (O'Connor & Robinson, 2014). She wrote her thesis on "The extension of the Galois theory to linear differential equations" (Riddle, 2016). She graduated with a Master's degree in 1904 (Riddle, 2016). While there, she won a scholarship to Radcliffe College (O'Connor & Robinson, 2014). Radcliffe College was a woman's college with classes taught by professors from Harvard University (Mehren, 1999). It is now part of Harvard University (Mehren, 1999). She spent the next year studying math at Radcliffe. Studying at Radcliffe allowed Johnson to attend lectures by leading mathematicians and scientists such as Maxime Borcher, William Osgood, and Charles L Bouton (Riddle, 2016; O'Connor & Robinson, 2014). These lectures had a large impact on Johnson's interests in mathematics. Borcher was a prolific mathematician known for his work in differential equations ('Maxime Borcher', 1918). Osgood was a leading function theorist and analyst (Zund, 2010). Bouton, a man best known for his paper analyzing the mathematical properties of the game of "Nim", had the smallest influence on Johnson (Bouton, 1901). She earned her second master's degree in 1905 (Riddle, 2016).

While she was studying at Radcliffe, Johnson won the Alice Freeman Palmer Fellowship from Wellesley College, enabling her to study at Göttingen University in Germany from 1906-1907 (Riddle, 2016). The Alice Freeman Palmer Fellowship was awarded annually to a female graduate of an American college (Grinstein & Campbell, 1982, pg. 39). Johnson was one of five women who applied (Grinstein & Campbell, 1982, pg. 39). While in Göttingen, Johnson attended lectures by internationally acclaimed mathematicians such as David Hilbert, Felix Klein, Hermann Minkowski, Gustav Herglotz, and Karl Schwarzschild (Riddle, 2016; O'Connor & Robinson, 2014). Klein was best known for his work on systems of non-Euclidean geometry while Minkowski is best known for utilizing geometric methods in the theory of numbers (Jones, 2017; 'Hermann Minkowski', 2016). Herglotz was a prolific mathematician who worked in geometry, differential equations, and function theory, while Schwarzschild an accomplished astrophysicist (Barile & Weisstein, 2007; Alt, 2017). Hilbert is the most accomplished mathematician in this group, and he had the most interaction with Johnson while she studied at Göttingen (Riddle, 2016). Hilbert was a proponent of rigor and formalism best known for his 23 fundamental questions (Franceschetti, 2015). The field of integral equations was active in the early twentieth century, and Hilbert's influence particularly helped Johnson narrow down an area in which to conduct her research (Grinstein & Campbell, 1982, pg. 39). In this year, Hilbert advised Johnson as she worked toward her doctorate (O'Connor & Robinson, 2014).

When her fellowship ended in June 1907, Johnson married Alexander Pell in July 1907 in Göttingen despite the objections of her family due to the 25-year age gap between them (O'Connor & Robinson, 2014). Pell described Johnson as being "something like a demi-goddess now, for whatever she wants she gets and whatever she studies she make a success of" (O'Connor & Robinson, 2014). They married immediately after the end of Johnson's fellowship because the Alice Freeman Palmer Fellowship did not allow the holder to be married (O'Connor & Robinson, 2014). They returned to the University of South Dakota that fall where Pell was the Dean of Engineering (Riddle, 2016; O'Connor & Robinson, 2014). That fall, Johnson Pell taught two mathematics courses at the university, one on the theory of functions, and another on differential equations (Riddle, 2016; O'Connor & Robinson, 2014).

Johnson Pell returned to Göttingen the following spring to work on her doctoral thesis (Riddle, 2016). She spent most of 1908 in Göttingen without her husband, completing most of the work for her doctorate (O'Connor & Robinson, 2014). However, Johnson Pell left that December without her degree and moved to Chicago, where Pell had taken a faculty position (O'Connor & Robinson, 2014). His new position was at the Armour Institute of Technology, now known as the Illinois Institute of Technology (O'Connor & Robinson, 2014). There is no record of exactly what transpired. Johnson Pell had been making preparations as late as November (Grinstein & Campbell, 1982, pg. 40). Johnson Pell's father wrote to her sister that he believed that Pell had pressured her into coming home early (Grinstein & Campbell, 1982, pg. 41). Johnson Pell told her friend, Mary Coes, Dean of Radcliffe, that it was because she "had some trouble with Professor Hilbert" (Grinstein & Campbell, 1982, pg. 41; Riddle, 2016). No record exists divulging the substance of the disagreement between Johnson Pell and Hilbert (Riddle, 2016).

Johnson Pell immediately joined the University of Chicago's graduate program in January of 1909 (Riddle, 2016). In addition to studying with well-known astronomers and mathematicians, Johnson Pell studied with the chair of the mathematics department, Eliakim Moore (Grinstein & Campbell, 1982, pg. 41; Riddle, 2016). In the spring and the fall semesters she studied general analysis, periodic orbits, number theory, integral equations, the theory of algebraic numbers, and "Modern Analysis applied to celestial Mechanics", as well as integral equations in general analysis (O'Connor & Robinson, 2014; Grinstein and Campbell, 41). In between semesters she worked at the Chicago Observatory (O'Connor & Robinson, 2014). While she studied, she developed an interest in "linear algebra of infinitely many variables", an area of functional analysis (Riddle, 2016). Johnson Pell received her Ph.D. with a thesis on biorthogonal systems of functions in 1909 (Riddle, 2016). This thesis was, for the most part, the thesis that she had written in Göttingen (Riddle, 2016). Johnson Pell wrote Mary Coes that she "had a right to use" her thesis at the University of Chicago because it had been "written independently of Hilbert" (O'Connor & Robinson, 2014). She received her degree magna cum laude (O'Connor & Robinson, 2014). Johnson Pell was the second woman to receive a PhD in mathematics at the university, and the first under Prof. Moore (O'Connor & Robinson, 2014).

## Professional Work in America

After graduating, she published two papers between 1909 and 1910 in the *Bulletin* of the American Mathematical Society, one titled *On an Integral Equation with an Adjoined Condition*, and the other titled *Existence Theorems for Certain Unsymmetric Kernels* (O'Connor & Robinson, 2014). Moore attempted to help her find a teaching position at a university in the area, but had no success (Riddle, 2016). Johnson Pell noted in a letter to Mary Coes that she thought "Professor Moore ha[d] also given up hope for he ha[d] inquired at some of the Eastern Girls' Colleges and Bryn Mawr [was] apparently the only one with a vacancy in mathematics" (O'Connor & Robinson, 2014). She added the observation that there was "such an objection to women that they prefer a man even if he is inferior both in training and research" (Riddle, 2016). Unwilling to move away from her husband, Johnson continued to search for a job in the area and taught a course in the fall of 1910 at the University of Chicago (O'Connor & Robinson, 2014).

Johnson Pell's inability to find a job was not a consequence of her teaching capabilities. Johnson Pell had a reputation as a "fine teacher" (Grinstein & Campbell, 1982, pg. 42). The former president of the University of South Dakota wrote that Johnson Pell gave "all her mind and energy to her teaching and [was] always willing to assist individual students out of hours", and also described her as an "instinctively kind and interested" teacher who easily passed her enthusiasm for mathematics onto her students (Grinstein & Campbell, 1982, pg. 42).

Pell had a stroke the following spring so Johnson Pell stepped in at the Armour institute to teach his math classes (Riddle, 2016). She wrote to Coes that the math department at the Armour institute "were practically forced to take [her] for they could not get a man" (O'Connor & Robinson, 2014). The department was so satisfied with her work that they told Mr. Pell to take the rest of the semester to recover while Johnson Pell taught his five classes (O'Connor & Robinson, 2014). Johnson Pell wrote that her friends and colleagues at the University of Chicago "were very much pleased that at last a woman had the chance to show her ability in such a place as the Armour Institute" (O'Connor & Robinson, 2014). Despite this success, Johnson Pell knew that it would "take a great number of years, to break down the prejudice" (O'Connor & Robinson, 2014).

Around this time, Johnson Pell's doctoral thesis was published in two parts in the *Transactions of the American Mathematical Society,* Vol. 12 (1911) (Riddle, 2016). The first part of Johnson Pell's thesis "develops the theory of biorthogonal systems of functions independent to its connection with differential and integral equations" (Grinstein & Campbell, 1982, pg. 44). A biorthogonal system is a system such that "if $\{u_i\}$ and $\{v_j\}$ are subsets of $L^2$, the class of Lebesgue-square-integrable functions on the interval [a,b], then $\{u_i\}$ and $\{v_j\}$ form a biorthogonal system if and only if $\int u_i v_j = 1$ whenever i=j, and 0 otherwise," integrated from a to b (Grinstein & Campbell, 1982, pg. 44). These systems were first introduced by Murphy in 1833 (Grinstein & Campbell, 1982, pg. 44). Schmidt and Birkoff demonstrated that these systems arise as solutions of adjoint pairs of integral equations or differential equations (Grinstein & Campbell, 1982, pg. 45). Johnson Pell's thesis established the necessary and sufficient conditions for "associating an adjoint system with a system of linearly independent functions, gave conditions for a generalized principal axis theorem, $\int fg = \sum \int fv_i \int u_i g$, to hold, and classified biorthogonal systems into equivalence classes" (Grinstein & Campbell, 1982, pg. 45). Equivalence classes have a one-to-one correspondence with linear operators (Grinstein & Campbell, 1982, pg. 45). Several of the operators used corresponded to the Hilbert-Schmidt theory for symmetric kernels (Grinstein & Campbell, 1982, pg. 45). The bulk of her later results stem from this paper (Grinstein & Campbell, 1982, pg. 45). The paper was presented to the American Mathematical Society in September 1910 (Grinstein & Campbell, 1982, pg. 45).

The second part of Johnson Pell's thesis applied the theory from the first part to the theory of integral equations (Grinstein & Campbell, 1982, pg. 44). She utilized the principal axis theorem to obtain "an expansion theorem for functions in terms of solutions to an integral equation" (Grinstein & Campbell, 1982, pg. 45). In this paper, she showed that the "functional equation $f = \lambda T(f)$ has a solution exactly when the quadratic form corresponding to T has an eigenvalue" (Grinstein & Campbell, 1982, pg. 45). She also found that "if L is a kernel for which $M(s,t) = T_S L(s,t) = \int K(s,r)L(r,t)dr$ is symmetric, then L itself has all the desirable properties of a symmetric kernel" (Grinstein & Campbell, 1982, pg. 45).

It soon became clear that Pell was not going to recover enough from his stroke to teach again, so Johnson Pell took a job as an instructor at Mt. Holyoke College (Riddle, 2016; O'Connor

& Robinson, 2014). Mt. Holyoke was a women's college in South Hadley, Massachusetts (O'Connor & Robinson, 2014). Two years later, in 1914, Johnson received a promotion and became an associate professor (O'Connor & Robinson, 2014). The same year she published a paper on "non-homogeneous linear equations in infinitely many unknowns" (O'Connor & Robinson, 2014). This paper, however includes a false lemma, greatly diminishing its value (Grinstein & Campbell, 1982, pg. 47). In 1916, she worked with Ruth L. Gordon to publish *The Modified Remainders Obtained in Finding the Highest Common Factor of Two Polynomials* (O'Connor & Robinson, 2014). This paper "patches up an algorithm of Van Vleck's for calculating the greatest common denominator of two polynomials" (Grinstein & Campbell, 1982, pg. 47). This paper on Sturm's theorem includes a theorem on polynomial remainder sequences that they prove with induction (Akritas, 2013). This paper has been used as recently as 2013 to inspire the creation of new methods to compute polynomial remainder sequences (Akritas, 2013).

Johnson Pell chose to take a position as an associate at Bryn Mawr College in 1918 (Riddle, 2016). Bryn Mawr was a research oriented women's college in Pennsylvania (Hunger Parshall, 2015, pg. 71, 78). Johnson Pell picked Bryn Mawr over Mt. Holyoke College primarily because Bryn Mawr had a PhD program (O'Connor & Robinson, 2014). Johnson Pell also saw an opportunity to become chairwoman after Charlotte Scott retired (Grinstein & Campbell, 1982, pg. 43). Johnson Pell supervised eight PhD students in her time at Bryn Mawr (Riddle, 2016). These students were Margaret Buchannan, Marion Gray, Laura Guggenbuhl, Rose Anderson, Olive Hughes, Vera Ames, Dorothy Maharam, and Josephine Mitchell (Hunger Parshall, 2015, pg. 78). Buchannan was her first PhD student, writing a thesis on Systems of Two Linear Integral Equations with Two Parameters and Symmetrizable Kernels (Hunger Parshall, 2015, pg. 79). She taught several courses at Bryn Mawr as well, including courses on analysis and the theory of integral equations (Hunger Parshall, 2015, pg. 78). Pell died in 1921 (Riddle, 2016). In 1924, Charlotte Scott retired and Johnson Pell became the head of the Bryn Mawr mathematics department (Riddle, 2016). The following year she was named Alumnae Professor of Mathematics (O'Connor & Robinson, 2014). Later that year, she married Arthur Wheeler, a colleague at Bryn Mawr (Riddle, 2016).

**Later Career**

When Arthur Wheeler was named professor of classics at Princeton University, Johnson Pell Wheeler moved to Princeton with her new husband, leaving her faculty position at Bryn Mawr in 1927 and becoming a part-time lecturer (Hunger Parshall, 2015, pg. 79). She published L*inear Ordinary Self-Adjoint Differential Equations of the Second Order* in 1927 (O'Connor & Robinson, 2014). While living in Princeton, Johnson Pell Wheeler continued to work on her research and interacted with the mathematicians at Princeton and at the Institute for Advanced Study (Hunger Parshall, 2015, pg. 79). Johnson Pell Wheeler also took a job as Associate Editor of the *Annals of Mathematics*, the second oldest research journal in America (Hunger Parshall, 2015, pg. 79). She was an editor there for the next eighteen years (Riddle, 2016). Johnson Pell Wheeler and her husband built a summer home in the Adirondack mountains which they named "Q.E.D." (O'Connor & Robinson, 2014).

In 1931, Johnson Pell Wheeler resumed her full-time professorship at Bryn Mawr and became department chair again after her husband's 1932 death (Hunger Parshall, 2015, pg. 79). She remained the department chair until her retirement in 1948 (Riddle, 2016). One of her first actions was to work with William Flexner, Gustav Hedlund, and Marguerite Lehr to drastically expand the graduate program, which now included a greater emphasis on research (Hunger Parshall, 2015, pg. 79). Another change included the addition of nearly seven hours a week of seminars (Hunger Parshall, 2015, pg. 79). Johnson Pell Wheeler gave seminars on complex analysis (Hunger Parshall, 2015, pg. 79). In 1933-1934, she continued to work to expand the Bryn Mawr graduate program with a new cooperative program between Bryn Mawr, the University of Pennsylvania, Haverford, and Swarthmore which involved the exchange of faculty, cross-institutional borrowing of books, and coordinated library purchases (Hunger Parshall, 2015, pg. 79). The program was aimed to reduce costs during the Great Depression without reducing educational quality (Hunger Parshall, 2015, pg. 79). Johnson Pell Wheeler was one of the first professors to give a lecture at one of the other colleges, presenting a seminar on linear functional equations (Hunger Parshall, 2015, pg. 79). Throughout her tenure at Bryn Mawr, Johnson Pell Wheeler strove to create an atmosphere that encouraged intellectual stimulation (O’Connor & Robinson, 2014).

One of Johnson Pell Wheeler’s major accomplishments as an administrator was in bringing Emmy Noether to Bryn Mawr in 1933 (Riddle, 2016; O’Connor & Robinson, 2014). Emmy Noether was a German-Jewish mathematician who was one of the founders of abstract algebra and a prominent theoretical physicist considered by many to be the most important woman mathematician in the world at that time (Caulfield, 2017). When life became unsafe for Noether in Germany, Johnson Pell Wheler played a crucial role in getting Noether professional and political asylum at Bryn Mawr College in an appointment that would allow Noether to work with graduate students and continue her research (Grinstein & Campbell, 1982, pg. 49). Noether died suddenly following surgery in 1935 (Grinstein & Campbell, 1982, pg. 49). Noether was one of Johnson Pell Wheeler’s close friends. Her death capped of a fifteen-year period of significant losses for Johnson Pell Wheeler, who lost both parents, two husbands and Noether in that time (O’Connor & Robinson, 2014).

Johnson Pell Wheeler’s activity in mathematics extended beyond involvement in the college. She formulated guidelines for college entrance examinations as part of a 1933-1935 College Entrance Examination Board Committee (Grinstein & Campbell, 1982, pg. 47). She was an active member of the American Mathematical Society, serving on the Board of Trustees and the Council (Riddle, 2016). In 1939, she was appointed to a committee dealing with “the use of accrued income from special funds” (Grinstein & Campbell, 1982, pg. 47). That same year, she supervised a meeting on analysis (Grinstein & Campbell, 1982, pg. 47). She was also an active member of the Mathematical Association of America (Riddle, 2016). There she served “on a three-person committee to select the winner of the first Chauvenet prize for excellence in mathematical exposition” (Grinstein & Campbell, 1982, pg. 47). She was also the chairwoman of the Mathematical Association of America’s Philadelphia section from 1943-1944 (Grinstein & Campbell, 1982, pg. 47). In 1939, she and other leading mathematicians petitioned to create an American analog of *Zentralblatt fiir Mathematik und ihre Grenzgebiete*, a German abstract and review journal (Grinstein & Campbell, 1982, pg. 47).

After her 1948 retirement, Johnson Pell Wheeler corresponded with her former students and attended mathematics meetings (Riddle, 2016). She spent her retirement traveling and enjoying the outdoors at Q. E. D (Grinstein & Campbell, 1982, pg. 49). She died on March 26, 1966 at the age of 82 at her Bryn Mawr home from complications following a stroke (Riddle, 2016; O’Connor & Robinson, 2014).

## A Closer Look at Her Main Contributions

Anna Johnson Pell Wheeler was a trailblazer for women in mathematics. In 1923, Johnson Pell Wheeler was the first woman to deliver an invited address at a meeting of the American Mathematical Society (O’Connor & Robinson, 2014). There she was named one of the 31 original trustees of the AMS and presented an address entitled "Bilinear and Quadratic Forms in Infinitely Many Variables (Hunger Parshall, 2015, pg. 79). More famously, in September, 1927, she became the first woman to deliver the Colloquium Lectures at the American Mathematical Society (Grinstein & Campbell, 1982, pg. 47; Riddle, 2016). She was the only woman to do so until Julia Robinson in 1980 (Riddle, 2016). Her colloquium lectures were on the theory of quadratic forms in infinitely many variables and its applications (Grinstein & Campbell, 1982, pg. 47). These lectures "summarized and surveyed the broader scene in which her own work had contributed during the previous 20 years" (Grinstein & Campbell, 1982, pg. 47).

Johnson Pell Wheeler worked primarily in functional analysis and its applications to differential and integral equations (Grinstein & Campbell, 1982, pg. 44). She worked on these equations and studied infinite dimensional linear spaces when functional analysis was undeveloped and inchoate (O'Connor & Robinson, 2014). As the field developed, the importance of her work diminished as it was absorbed into the general theory (O'Connor & Robinson, 2014). Her research was heavily influenced by Hilbert (O'Connor & Robinson, 2014). She also worked on the theory of quadratic forms in infinitely many variables and applications (Riddle, 2016). Johnson Pell Wheeler's other work includes an investigation on the "spectrum of a special real matrix" (Grinstein & Campbell, 1982, pg. 47). She presented many papers that were not published (Grinstein & Campbell, 1982, pg. 47).

Specifically, she investigated the Fredholm equation of the second kind, a linear integral equation, $u(s)= f(s) \lambda\int K(s,t)\ u(t)\ dt$, "where f and K are known continuous functions, $\lambda$ is a parameter, and the unknown is the continuous function u, and the function K is the kernel of the equation" (Grinstein & Campbell, 1982, pg. 44). She developed theorems that covered the general case of $L^2$ functions (Grinstein & Campbell, 1982, pg. 44). She utilized biorthogonal systems and operators as tools to extend the results of Hilbert, Schmidt, and Fredholm to "a wider class of kernels, including those now known as 'symmetrizable'" (Grinstein & Campbell, 1982, pg. 44). Her 1919 paper, *Linear equations with unsymmetric systems of coefficients*, generalized the work from her doctoral thesis to infinite linear algebraic systems using "a biorthogonal system of functions to reduce a given system of equations to one with a symmetric (infinite) matrix of coefficients" (Grinstein & Campbell, 1982, pg. 45). The main theorem of this paper asserts that if A is a limited matrix with a positive definite limited symmetric matrix T such that AT is symmetric, then A has a nonempty set of real eigenvalues (Grinstein & Campbell, 1982, pg. 45). Johnson Pell Wheeler applied this theorem to a theorem on the Radon Integral in a second 1919 paper, *A general system of linear equations* (Grinstein & Campbell, 1982, pg. 47). She used it in a 1922 paper, and in a 1927 paper to "establish a direct correspondence between linear algebraic equations and linear differential systems of the second order" (Grinstein & Campbell, 1982, pg. 47). Many of her students have used the theorem in their graduate work (Grinstein & Campbell, 1982, pg. 47).

**Legacy**

Anna Johnson Pell Wheeler has a respectable legacy in mathematics. In 1921, Anna Johnson Pell Wheeler received a star next to her name in the third edition of *American Men of Science*, "indicating that she was considered prominent among American mathematicians" (Grinstein & Campbell, 1982, pg. 47). She received an honorary doctorate from The New Jersey College for Women, which is now known as Douglass College, Rutgers University, in 1932, and one from Mt. Holyoke College five years later (O'Connor & Robinson, 2014). She was named as one of the "100 American women to have succeeded in careers not open to women a century before" by the Women's Centennial Congress in 1940 (Riddle, 2016).

Johnson Pell Wheeler was a highly praised and dedicated teacher (Grinstein & Campbell, 1982, pg. 48). It was said that she "gave generously of her time, her money, and herself to her students," offering needy students "copies of books she claimed she no longer used" just before new copies would appear on her shelves (Grinstein & Campbell, 1982, pg. 48). Johnson Pell Wheeler took her graduate students to Q.E.D with her so they would have enough time to work on their research, and to "professional meetings at surrounding colleges and universities" (Grinstein & Campbell, 1982, pg. 48). She was widely praised by her students. One student remarked that though her parents had initially been wary of their daughter pursing mathematics, meeting Johnson Pell Wheeler had completely changed their minds (Riddle, 2016). Another student noted that Johnson Pell Wheeler gave most of her time to her students, listening to them and assisting them with their "emotional and philosophical growth" as well as their growth in mathematics (O'Connor & Robinson, 2014).

Johnson Pell Wheeler was celebrated during her retirement and after her death as well. In 1960, an anonymous donor established the Anna Pell Wheeler prize at Bryn Mawr (O'Connor & Robinson, 2014). Four years later, a mathematics graduate seminar room was named after her (Grinstein & Campbell, 1982, pg. 48). Her life, career, and legacy was celebrated at a symposium held on August 20, 1980, at Ann Arbor, Michigan, featuring relatives and former students (Case, 1982, pg. 4).

References

Akritas, A. G (2013). *Various New Methods for Computing Subresultant Polynomial Remainder Sequences (PRS's)*. Retrieved from http://math.unm.edu/~aca/ACA/2015/Education/Akritas.pdf.

Alt, A. L. (2017). Karl Schwarzschild. *Salem Press Biographical Encyclopedia*,

Barile, M., & Weisstein, E. W. (2007). *Herglotz, Gustav (1881-1953)*. Retrieved from http://scienceworld.wolfram.com/biography/Herglotz.html.

Boman, E. J. (2001). Anna Pell Wheeler (b. Johnson) (1883-1966). Cambridge Dictionary Of American Biography, 1.

Bouton, C. L. (1901). Nim, A Game with a Complete Mathematical Theory. *Annals of Mathematics*, (1/4). 35.

Case, B. A. (1982). Anna Johnson Pell Wheeler: Colloquium Lecturer, 1927. Newsletter of the AWM, 12 (4), pg. 4-13.

Caulfield, M. J. (2017). Emmy Noether. *Salem Press Biographical Encyclopedia*.

Franceschetti, D. R. (2015). David Hilbert. *Salem Press Biographical Encyclopedia*.

Grinstein, L. S., & Campbell, P. J., (1982). Anna Johnson Pell Wheeler: Her life and work. *Historia Mathematica*, *9*(1), pg. 37-53. https://doi.org/10.1016/0315-0860(82)90136-7.

'Hermann Minkowski'. (2016). *Columbia Electronic Encyclopedia, 6th Edition*, 1.

Hunger Parshall, K. (2015). Training Women in Mathematical Research: The First Fifty Years of Bryn Mawr College (1885-1935). *Mathematical Intelligencer*, *37*(2), 71-83. http://doi.org/10.1007/s00283-015-9540-2.

Jones, A. R. (2017). Felix Klein. *Salem Press Biographical Encyclopedia*.

'Maxime Bocher'. (1918). *Science*, *48*(1248), 534-535. Retrieved from http://www.jstor.org/stable/1642928.

Mehren. (1999, Apr 21). *End of an Era: Harvard and Radcliffe Now Officially One*. *Los Angeles Times*. Retrieved from http://articles.latimes.com/1999/apr/21/news/mn-29517.

O'Connor, J.J., & Robertson, E. F. (2014, July). *Anna Johnson Pell Wheeler*. Retrieved from http://www-groups.dcs.st-and.ac.uk/~history/Biographies/Wheeler.html.

Riddle, L. (2016, Feb 25). *Anna Johnson Pell Wheeler*. Retrieved from https://www.agnesscott.edu/lriddle/women/wheeler.htm.

Zund, J. D. (2010). William Fogg Osgood. *American National Biography (From Oxford University Press).*