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## Father and Son Duo Etienne and Blaise Pascal

There have been many famous French mathematicians in history that have made great contributions to further develop and thus revolutionize the field of mathematics. One of which is Etienne Pascal and later on his genius son Blaise Pascal. Etienne and Blaise have accounted for many topics and theorems in mathematics that are used and applied today. Etienne's credibility is his participation in the committee to prove Jean-Baptiste Morin's claims, his work with the limacon and a special case of the limacon known as the cardioid. Blaise, of course was known for the famous Pascal's Triangle, which led to convention for the binomial theorem, and thus led to the beginning of the mathematical theory of probabilities.

Etienne Pascal lived from his birth on (May 2, 1588), in Clermont, Auvergne, France (now known as Clermont-Ferrand) until his death on September 24, 1651 (63 years). His father Martin Pascal was the treasurer of France, so he grew up in a wealthy family that had an ample amount of money, and he did not really need to work to earn a living. Even though he had a great family background he still obtained a law degree in Paris in 1610, and sought after influential jobs that would help service the French nation. Etienne served as a tax official, lawyer and a member of the petite noblesse. During his time in the French government, Etienne was in a certain degree of danger of assassination, as French peasants were revolting against the French government. The country was led by a nine-year-old King Louis XIII after he was handed the throne after his father King Henry IV's assassination. Etienne married Antoinette Begon in 1616, leading to the birth of his three children Gilberte, the eldest, in 1620,

Jacqueline, the youngest, in 1625 and of course Blaise, the middle child, in 1623. Etienne's work in the government led to his main contribution in history with him being selected to a committee in 1634 to prove Morin's claims that determining longitude from the moon's motion was practical.

The background of Morin's claims came from the $16^{\text {th }}$ century when Phillip II and Phillip III offered a prize of 6000 ducats plus 2000 ducats life income with the 1000 ducats expenses that was offered, to "the discoverer of longitude." Later on Holland offered a larger prize to "the inventor of a reliable method of finding the longitude at sea." In 1634 Jean-Baptiste Morin made a claim in his own country of France. Morin did not believe in the transporting clock method proposed by Gemma Frisius, saying "I do not know if the Devil will succeed in making a longitude timekeeper but it is folly for man to try." To convince Etienne and the others on the committee, Morin used the lunar distance method and took into account lunar parallax, setting up an observatory to gather and to furnish more accurate lunar data. Morin was awarded 2000 livres for his efforts in 1645 despite the committee thinking his method was still not practical and the two sides disputed back and forth for five years.

Etienne's main contribution to mathematics was the limacon of Pascal, or more simply known as the limacon. The limacon was first discovered by Etienne but was named by another French mathematician Gilles-Personne Roberval in 1650, when the limacon was used as an example in Roberval's methods of drawing tangents in differentiation. The limacon is an anallagmatic curve, meaning that is invariant under inversion, the equation for the limacon is given in the Cartesian coordinates as $\left(x^{2}+y^{2}-2 a x\right)^{2}=b^{2}\left(x^{2}+y^{2}\right)$ and the equation in polar coordinates is given by $r=b+2 a \cos (\theta)$. The name limacon comes from the Latin word limax
meaning "a snail." Before Etienne, Albrecht Durer, a German mathematician gave a method to draw the limacon. In 1826, another Frenchman Thomas de St. Laurent, showed that the limacon is also the catacaustic of a circle when light rays come from a point that is a non-zero, finite distance from the circumference.

## Limaçon of Pascal



## Cardioid



The cardioid is a special case of the limacon with equations given on Cartesian coordinates as $\left(x^{2}+y^{2}-2 a x\right)^{2}=4 a^{2}\left(x^{2}+y^{2}\right)$ and polar coordinates as $r=2 a(1+\cos (\theta))$. The cardioid also can be given parametric equations, such as $x=a(2 \cos (t)-\cos (2 t)), y=a(2 \sin (t)-$ $\sin (2 t))$. The name cardioid was first used by an Italian mathematician Johann Castillion in 1741, meaning 'heart-shaped'. The cardioid is a curve that is the locus of a point in the circumference of a circle rolling around the circumference of a different circle of equal radius. The length, given as 16a with the notation used, was founded in 1708 by French mathematician Philippe de la Hire. Also at any given gradient, there are three parallel tangents to the cardioid,
with the tangent at the ends of any cord through the cusp forms a right angle and the cords having a length of $4 a$ and an area of $6 \pi a^{2}$.

Moving onto the son of Etienne, Blaise Pascal was considered a child prodigy and is much better known for his contributions to the intuitions of mathematics than his father. Blaise was born on June 19, 1623 and lived until August 19, 1662 (only 39 years compared to his father's 63 years). Blaise was born in the same city and region as his father, but moved to Paris at only eight-years old. He eventually worked as a mathematician, physicist, inventor, writer, and Christian philosopher. Growing up taught by Etienne, Blaise was actually not allowed to study mathematics until the age of fifteen and all of the mathematical texts they had were removed from the house. As with every rebellious child, the more that parents say no, the more interest is generated by the child, and thus Blaise began working on geometry at age twelve. He discovered that the sum of the angles in a triangle are two right angles, and when his father found out, Etienne took a step back and allowed him to study Euclid. At age fourteen Blaise accompanied his father to Mersenne's meetings and at age sixteen, in June 1639, he presented a single piece of paper at one of the meetings that contained a number of theorems in projective geometry including Pascal's mystic hexagon (image at the end).

In 1645, Pascal invented a mechanical calculator called the Pascaline after working on it for three years; the calculator was designed to help his father with tax collection. From 1642 when he began working on the calculator, there had already been fifty prototypes created before the model was presented to the public in 1645 . Though the first mechanical calculator was manufactured in 1624 by a German astronomer Wilhelm Schickard, Pascal's was more practical and paved the way for the development of future calculators. Due to the French
currency at the time where 20 sols were in a livre and 12 deniers were in a sol, Blaise had to solve more difficult technical problems since the livre had to be divided by 240 rather than 100 . With this problem in effect, by 1652, ten years after production started, very few were sold, and thus production ceased that year.

After his father Etienne's death in 1651, Blaise worked on mathematics and physics simultaneously. Pascal's work on physics focused on the study of hydrodynamics and hydrostatics centered around the principles of hydraulic fluids. From 1647-1648, he established the law known as both Pascal's law and Pascal's principle, defined as "a change in pressure at any point in an enclosed fluid at rest is transmitted undiminished to all points in the fluid." The mathematical equation is given as $\Delta P=\rho g(\Delta h)$, where $\Delta P$ is the hydrostatic pressure, $\rho$ is the density of the fluid, $g$ is acceleration due to gravity, and $h$ is the height of the fluid at the point of measurement. This principle led to the inventions of the hydraulic press and the syringe. Also in 1647, Pascal experiments followed up on Aristotle's claim that "everything in motion must be moved by something." In his "New Experiments with the Vacuum" of 1647, he explained basic rules in regards to what degree various liquids can be supported by air pressure and provided reasoning as to why a vacuum was above the column of liquid in a barometer tube. In 1653, he wrote the "Treatise on the Equilibrium of Liquids" in which he explains Pascal's law of fluid pressure. Suppose that there is a U-tube that is filled with water, and that there are pistons at each end. If pressure is exerted against the left piston, the pressure is transmitted through the water until it reaches the right piston. The amount of pressure on each piston is equal to the other. In another example if the ends of the tube are not equal and a larger piston is used on one end, the difference between force and pressure is pivotal. The additional pressure will be
exerted against the entire area of the larger piston, allowing the larger one to support a load equal to the ratio between the sizes of the two pistons (i.e. if the larger one has 10 times the area of the smaller one, then it can support a load 10 times the force of the smaller one).

Blaise Pascal's work in mathematics also includes the beginning of a new topic known as the mathematical theory of probability, that he developed with fellow French mathematician Pierre de Fermat. The theory of probability began with the subject of a gambling problem. The question posed by a friend of Pascal, Chevalier de Mere, pertained to if two fair six-sided dice are rolled, what would be the number of turns required to ensure that a six is rolled. The circumstances of the game are as follows; both players wanted the game to finish as early as possible, wanted to divide stakes fairly, given the chance that both have to win the game from that point. With this, the notion of expected value was instituted. Pascal later used a probabilistic argument to justify belief in God and a virtuous life; the argument is called Pascal's Wager.

Between Blaise and Etienne Pascal, they have combined for many great accomplishments in mathematics and physical science, but the most well-known element is Pascal's Triangle. For many people, the first thing that comes to mind when they hear the name Pascal is Pascal's Triangle. The first five rows of Pascal's Triangle are 1, $11,121,1331$, and 1 464 1, respectively. For the $n^{\text {th }}$ row in the triangle, that row has $n$ number of terms (i.e. the fifth row has five terms). The first and last term of every row is 1 , while terms in between are obtained by recursion from adding two numbers directly above it. For example, the second term in the sixth row (5) is obtained from $1+4$ in the previous row, the third term in the sixth row (10) is obtained from adding $4+6$, and so on. Another point of interest in the triangle is
that the sum of each row is equal to $2^{n}(n=0,1,2, \ldots, n)$. The sum of the first row is $1=2^{0}$, the second is equal to 2 which is $2^{1}$, the third is equal to 4 which is $2^{2}$, and it keeps repeating in that pattern infinitely. Also each row represents the powers of 11 , since the first row $1=11^{0}$, the second row 11 or 11 is equal to $11^{1}$, the third row 121 or 121 is equal to $11^{2}$, and this pattern also repeats until the fifth row $\left(11^{4}\right)$. From the fifth row on, the digits just overlap, $11^{5}=161051$ and the terms form the sixth row of the triangle are 1510105 1. The 10's are split and overlap with the previous thus they are added, so the second digit is $6=5+1$, the third is $0+1=1$, then 0 , and the two end digits are the same, hence $11^{5}=161051$ is obtained.

The most important application of Pascal's triangle is its connection to the theory of probability. If a fair coin is tossed $n$ times, then the $(n+1)^{\text {th }}$ row of Pascal's triangle is used to show number of different combinations of outcomes and how many of each contain $x$ number of head and $y$ number of tails ( $x, y<n$ ). For example, it the coin is tossed once, there is only one possible way to get heads and one way to get tails, as the second row of Pascal's Triangle is 11. Another example is if the coin is tossed three times, then the possible outcomes are $\mathrm{HHH}, \mathrm{HHT}$, HTH, THH, HTT, THT, TTH, and TTT ( $\mathrm{H}=$ heads and $\mathrm{T}=$ tails). The fourth row of the triangle is 13 31 , which represents how many possible options there are to get all heads (1), two heads and one tails (3), two tails and one heads (3), and all tails (1). Another aspect of probability that the triangle plays a role in is combinations, including a formula for any term in the triangle, which is the formula $(n$ choose $k)=n!/ k!(n-k)$ !. If $n=0,1,2, \ldots$ then for every $n$, that $n$ represents the $(\mathrm{n}+1)^{\text {th }}$ row and let $\mathrm{k}=0, \ldots, \mathrm{n}$. If $\mathrm{n}=0$, means that term in the first row is $(0$ choose 0$)=1$, second row has $(1$ choose 0$)=1$ and $(1$ choose 1$)=1$, third row has $(2$ choose 0$)=1$, ( 2 choose 1 ) $=2$, and ( 2 choose 2 ) $=1$. Evaluating each of the combinations using the formula
give the terms in Pascal's triangle. There are also several other uses and applications associated with Pascal's Triangle, as it is one of the most famous phenomena of mathematics, and are still used today in many different areas of mathematics.

Mathematics is a language that is often considered the universal language, since it is the same in every country. In the United States $1+1=2$ and it does not change in China, Europe, or anywhere else in the world. The Pascal's, both Etienne and Blaise, have been involved with many aspects in the history of both mathematics and also in some aspects of physics. The father-son duo pioneered many ideas into mathematics that revolutionized understanding of concepts back then and led to an even better way to teach mathematics today. Blaise Pascal's work in physics helped lead to the invention of syringes, which are used today to save lives. With all of the work they have done, Etienne and especially Blaise Pascal are two of the greatest mathematicians and scientists to ever live.


## Pascal's Mystic Hexagram

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