Solutions to Attendance Quiz # 8 for Dr. Z.'s MathHistory for Lecture 8

1. In a certain country there are only two kinds of coins. One is worth 10 dollars each, and the other is worth 13 dollars each. How would you pay for a cup of coffee that costs exactly 1 dollar (without tipping!). What do you have to give to the cashier, and what does the cashier return to you?

Sol. of 1.

We have to find the greatest common divisor of 10 and 13, that is obviously 1, and at the same time express 1 as a combination of 10 and 13.

$$\mathbf{13} = 1 \cdot \mathbf{10} + 3$$

So $gcd(13, 10) = gcd(10, 3)$, and

$$3 = \mathbf{13} - \mathbf{10} \quad .$$

Now

$$\mathbf{10} = 3 \cdot 3 + 1$$

So the remainder is 1 and $gcd(10, 3) = gcd(3, 1) = 1$. We have

$$1 = \mathbf{10} - 3 \cdot 3 = \mathbf{10} - 3(\mathbf{13} - \mathbf{10}) = 4 \cdot \mathbf{10} - 3 \cdot \mathbf{13} \quad .$$

We have expressed 1 as a combination of **10** and **13**.

Now we can apply it to paying a cup of coffee that costs 1 dollar.

Hand the cashier four ten-dollar coins, and expect the cashier to give you a change of three thirteen-dollar coins.