Attendance Quiz # 19 for Dr. Z.'s MathHistory Lecture 20

NAME: (print!) _____

E-MAIL ADDRESS: (print!) _____

1. Using a calculator (or Maple, or any software), find the first 20 terms of the sequence defined by the recurrence.

$$x_{n+1} = a\, x_n(1 - x_n) \quad ,$$

starting at $x_0 = 0.6$, and decide whether it tends to a single number, or more of them. In the latter case, how many?

(i) When $a = 2.1$.

(ii) When $a = 3.1$

Sol.

(i)

The trajectory for $a = 2.1$ staring at $x_0 = 0.6$, repeatedly applying $x \to ax(1-x)$, i.e.

$$x \to ax(1-x)$$

is (using a calculator or, better a computer, once you hard-wired $f(x) = 2x(1-x)$ is

$$0.6, 0.504, 0.5249664, 0.5236910256, 0.5238213443, 0.5238083415, 0.5238096420, 0.5238095119,$$

$$0.5238095250, 0.5238095235, 0.5238095237, 0.5238095239, 0.5238095237,$$

$$0.5238095239, 0.5238095237, 0.5238095239, 0.5238095237, 0.5238095239, 0.5238095237, 0.5238095239,$$

$$0.5238095237, 0.5238095239, 0.5238095237, 0.5238095239 \quad .$$

It seems to converge to **one** fixed point.

(ii)

The trajectory for $a = 3.1$ staring at $x_0 = 0.6$, repeatedly applying $x \to ax(1-x)$, i.e.

$$0.6, 0.744, 0.5904384, 0.7496447770, 0.5818002046, 0.7542570521, 0.5745953896,$$

$$0.7577501364, 0.5690510883, 0.7602190365, 0.5650867644, 0.7618675108, 0.5624187609,$$

$$0.7629220848, 0.5607031297, 0.7635769031, 0.5596343702, 0.7639756000, 0.5589823361,$$

$$0.7642153605, 0.5585897543 \quad .$$

It seems to converge to an orbit of length 2.