NAME: (print!) _____

E-Mail address: _____

MATH 436 Exam II for Dr. Z.'s, Spring 2017, April 24, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

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1. (out of 10)

2. (out of 10)

3. (out of 10)

4. (out of 10)

5. (out of 10)

6. (out of 10)

7. (out of 10)

8. (out of 10)

9. (out of 10)

10. (out of 10)

11. (out of 10)

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total: (out of 110)

1. (10 pts.) Give **two** proofs of the Pythagorean theorem.

2. (10 pts.) Prove that $\sqrt[7]{3}$ is irrational.

3. (10 pts. total) (a) (5 points) Define the Mandelbrot set.

(b) (5 points) Define the Feigenbaum constant. Explain everything!

4. (10 pts. total)
(a) (6 pts.) For any position P in the (n^2-1)-puzzle, let $[i, j]$ be the location of the blank (that we call n^2) (In other words, i is the row-number and j is the column-number), and let $\pi(P)$ be the permutation of $\{1, 2, 3, \ldots, n^2\}$ obtained by reading it from left-to-right and top-to-bottom (like in English). Define

$$S(P) = i + j + \ inv\,(\pi(P)) \quad ,$$

where $inv\,(\pi)$ is the number of inversions of the permutation π.
Prove that if Q is any position reachable from P by a finite number of legal moves, then the **parity** of $S(P)$ equals the **parity** of $S(Q)$. In other words, they are either **both even** or **both odd**.
Note: You may use the lemma that if you exchange *any* **two** elements in a permutation, the number of inversions *always* changes by an odd integer.

(b) (4 pts)
Let

$$P = \begin{pmatrix} 1 & 2 & 3 \\ 5 & & 4 \\ 6 & 7 & 8 \end{pmatrix} \quad , \quad Q = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & \\ 6 & 7 & 8 \end{pmatrix} \quad .$$

Can you reach position P' from position P, by a sequence of legal moves? Explain!

5. (10 points)
Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x,y)+iv(x,y)$ where $u(x,y), v(x,y)$ satisfy the above system of two equations?

7. (10 points) Who discovered the quaternions? What city did that person live in?

8. (10 points) What is Heron's formula, what century did Heron live in?

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

11. (10 points total)
(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

(b) (5 points) State the names of two people who initiated the use of logarithms