NAME: (print!) \_\_\_\_\_

E-Mail address: \_\_\_\_\_

MATH 436 Exam I for Dr. Z.'s, Spring 2017, March 23, 2017, 10:20-11:40am, SEC 211

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTE-BOOK (But not your Math Notebook) Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

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- 1. (out of 10)
- 2. (out of 10)
- $3. \qquad (out of 10)$
- $4. \qquad (out of 10)$
- $5. \qquad (out of 10)$
- 6. (out of 10)
- 7. (out of 10)
- $8. \qquad (out of 10)$
- 9. (out of 10)
- 10. (out of 10)
- 11. (out of 10)

total: (out of 110)

1. (10 pts.) Prove that there are infinitely many primes.

**2.** (10 pts.) Prove that  $\sqrt{5}$  is irrational.

**3.** (10 pts.) Prove that

$$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{2n+1} \quad .$$

4. (10 pts.) Express

$$a(n) := \sum_{k=1}^{n} k(k-1)$$
 ,

as a polynomial of degree three in n. Prove it!

5. (10 points) Construct a seven by seven Magic Square.

**6.** (10 points) Arrange the following people according to their year-of-birth, from oldest to youngest.

Newton, Archimedes, Gallileo, Euler, Gauss, Zeilberger, Euclid, Thales, Brahmagupta, Fibonacci.

For each person, state their century of birth.

7. (10 points). What is an Egyptian fraction? Express  $\frac{5}{6}$  as an Egyptian fraction

8. (10 points) What is the difference between Ionian (Greek) mathematics and ancient Babylonian and Chinese mathematics? Who was the traditional father of Greek mathematics?

**9.** (10 points) What book, except for the bible, was the most reproduced and studied in the Western world? Who was its author?

10. (10 points) In a closed polyhedron, what is a relation between V, the number of vertices, E, the number of edges, and F, the number of faces? Who is it due to?

11. (10 points) What is the symbol, and name, of the following constant:

$$\lim_{n \to \infty} \left( \frac{1}{1} + \frac{1}{2} + \ldots + \frac{1}{n} - \log n \right)$$

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What is its approximate value?