Homework for Dr. Z.'s MathHistory for Lecture 4 (Due Feb. 16, 2017)

0. Read and understand Chapter III, sections 1-4 (pp. 39-50) summarize its content in your own words and your own handwriting, and write it in your HISTORY notebook, [You should have at least the equivalent of two typed pages, but you are welcome to write more]

The other problems should be in your MATH notebook (or loose sheets, whatever you prefer)

1. Two car-drivers, A and T compete in a race. A rides at a constant speed of 200 miles per hour, while T is much slower, and rides at a constant speed of 50 miles per hour.

To make it fair, T gets a head-start of 50 miles.

(a) Spell out Zeno's proof that A will **never** catch-up to T.

(b) Use high-school algebra (the formula that distance=speed multiplied by time) to predict at what time, in spite of Zeno, A will catch-up with T after all.

(c) Express the meeting time as an 'infinite' geometric series, and use the formula for summing an 'infinite' geometric series to get the same answer as (b).

(d) If, like Dr. Z. (and much greater people, like Hermann Weyl), you dislike 'infinite' sums, resolve the paradox, by assuming that the 'atom' (smallest unit) of time is $\frac{1}{64}$ of an hour. Compile a table of each successive step in the Zeno description of the race, but rather than going for ever, stop when it is no longer possible to sub-divide time.

2. Prove (from scratch) that, for any integer $n \geq 0$, and *any* number x,

$$1 + x + x^2 + \ldots + x^n = \frac{1 - x^{n+1}}{1 - x}$$

(Hint: one way is to use mathematical induction, a better way is to multiply both sides by $(1 - x)$ and see what happens.)

3. Prove that, if $0 < x < 1$, the value of the 'infinite' sum

$$\sum_{n=0}^{\infty} x^n$$

equals $\frac{1}{1-x}$.

(a) By using **2.** .

(b) By imagining that A runs 1 miles per second, and T runs x miles per second (where $x < 1$), and T gets a head-start of 1 mile in a race, and computing the meeting time in two ways. The first way, Zeno style, the second way, high-school algebra way.

4. Use the dichotomy paradox (Look it up in wikipedia) to prove that

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = 1 \quad .$$