Homework for Dr. Z.'s MathHistory for Lecture 15 (Due April 6, 2017)

Version of April 3, 2017 (Thanks to André Silva Diaz)

0. Read and understand Chapter VII, sections 8-10 (pp. 186-192), summarize its content in your own words, and your own handwriting, and write it in your HISTORY notebook, [You should have at least the equivalent of two typed pages, but you are welcome to write more]

1. Prove that in the fifteen puzzle, if you start with

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & \end{pmatrix}$$

It is **impossible**, by sliding, to get to the position

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 15 & 14 & \end{pmatrix} \quad ,$$

and hence that Sam Lloyd was safe in offering a large prize for its solution.

2. Define a group.

3. Prove that the set of all 2×2 matrices with integer entries, and determinant 1 is a group. **The operation is matrix-multiplication**. Is it a finite group?

In particular, (i) What is the identity element? (ii) how to find the inverse?

(Note: you do not have to prove the associativity, just say that it is known from Linear Algebra.)

4. (a bit of a challenge) Prove that the set of all 2×2 matrices with entries that are in $\{0, 1, 2\}$, with **non-zero determinant**, and where the operation is matrix multiplication done mod 3 is a group. Is it a finite group? How many elements does it have?

(Note: you do not have to prove the associativity, just say that it is known from Linear Algebra)

5. Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.