Homework for Dr. Z.'s MathHistory for Lecture 12 **(Due March 23, 2017)**

0. Read and understand Chapter VI, section 8 (pp. 154-160) summarize its content in your own words, and your own handwriting, and write it in your HISTORY notebook, [You should have at least the equivalent of two typed pages, but you are welcome to write more]

1. Use the definition

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n \quad ,$$

find the first three terms (i.e. $n = 0, 1, 2$) of the Taylor expansion around $x = 0$, of $f(x) = \sin(x+x^2)$

2. Using the known formula

$$\sin z = z - \frac{1}{3!}z^3 + \frac{1}{5!}z^5 - \frac{1}{7!}z^7 + \dots$$

and high-school algebra (pretending that the power series are polynomials), to find the first five terms (i.e. $n = 0, 1, 2, 3, 4$) of the Taylor expansion around $x = 0$, of $f(x) = \sin(x + x^2)$.

3. Using the fact that

$$\arctan x = \int_0^x \frac{1}{1+t^2}\, dt \quad ,$$

and the famous Taylor series for $\frac{1}{1-z}$ (the infinite geometrical series)

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n$$

derive the Taylor series (around $x = 0$) of the function $\arctan x$.

4. Using the trig-identity

$$tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - (\tan\alpha)(\tan\beta)} \quad ,$$

derive the arctan identity

$$\arctan x + \arctan y = \arctan \frac{x+y}{1-xy} \quad .$$