A GENERALIZATION OF ODLYZKO'S CONJECTURE:
THE COEFFICIENTS
OF \((1 - q)^j/((1 - q^{2n}) \cdots (1 - q^{2n+2j}))\) ALTERNATE IN SIGN

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ABSTRACT. A positivity result is proved that generalizes a conjecture of Odlyzko,
previously proved by Stanton and Zeilberger.

Let \(N, a, b, i, j, r, n\) denote arbitrary nonnegative integers, and let
\[
\begin{align*}
(x)_N & := \prod_{i=1}^{N-1} (1 - q^i x), \\
G(a, b) & := \frac{(q)_{a+b}}{(q)_a (q)_b}, \\
A & := \{ f(q); (-1)^i f^{(i)}(0) \geq 0, \ 0 \leq i < \infty \}.
\end{align*}
\]
Odlyzko conjectured that \((1 - q)^j/(q)_j \in A\). This was proved in [1], where the
more general fact that \((1 - q)^j G(2j, r) \in A\) was proved. Here we show that this
last result also implies the statement of the title, that it generalizes Odlyzko’s
conjecture (for even \(j\)) in a different direction. To wit: set \( t = q^{2n}, N = 2j\),
in the \(q\)-binomial theorem \( 1/(t)_{N+1} = \sum_{r \geq 0} G(N, r) t^r \), and then multiply by
\((1 - q)^j\). \(\square\)

REFERENCE

1. D. Stanton and D. Zeilberger, The Odlyzko conjecture and O’Hara’s unimodality proof,

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