with the property that no open connected subset of \( Y \) contains a cut point. Let \( A \) be a subset of \( X \) consisting of isolated points. Assume that the map \( f: X \to Y \) is continuous, and that \( f \uparrow X - A \) is an open map.

Then \( f \) is an open map.

Proof. It will suffice to show that \( f \) is open at \( x \) for each \( x \in A \). Assume that there is an \( x_0 \in A \) such that \( f \) is not open at \( x_0 \). Then there are open sets \( N \) and \( V \), with \( \overline{N} \) compact, such that \( x_0 \in N \subset \overline{N} \subset V \), and \( f(x_0) \not\in f(V)^0 \). Since the points in \( A \) are isolated, we may assume that \( V \cap A = \{ x_0 \} \). Then since \( f \uparrow X - A \) is an open map, \( f(V - \{ x_0 \}) \) is an open subset of \( f(V) \), so \( f(x_0) \not\in f(V - \{ x_0 \}) \).

Since \( \overline{N} - \{ x_0 \} \subset V - \{ x_0 \} \), \( f(x_0) \not\in f(\overline{N} - \{ x_0 \}) \), and \( f(N - \{ x_0 \}) \) is open. Since \( f \uparrow N \) is continuous and \( \{ x_0 \} \) is not an open subset of \( N \), \( f(x_0) \not\in f(N - \{ x_0 \}) \). On the other hand, \( f(\overline{N}) \) is closed, so
\[
\overline{f(N - \{ x_0 \})} \subset f(N - \{ x_0 \}) \cup f(\overline{N} - N) \cup \{ f(x_0) \},
\]
and since \( f(\overline{N} - N) \) is closed and does not contain \( f(x_0) \), it follows that \( f(x_0) \) is an isolated point in the boundary of the open set \( f(N - \{ x_0 \}) \). We have \( f(x_0) \not\in f(N - \{ x_0 \})^0 \), because \( f(N - \{ x_0 \}) \subset \overline{f(N)} \subset f(V) \), and \( f(x_0) \not\in f(V)^0 \). Therefore, (ii) in Lemma 1 is false for \( Y \), so (i) must also be false, contradicting our hypothesis. This concludes the proof of Theorem 1.

REFERENCES


On a Conjecture of R. J. Simpson About Exact Covering Congruences

DORON ZEILBERGER\(^1\)

Department of Mathematics, Drexel University, Philadelphia, PA 19104

The following is a counterexample\(^2\) to Simpson's conjecture [2]: \( D = \{6, 15, 35, 14, 210\) (140 times)\}. It was concocted using the elegant and powerful approach of [1].

REFERENCES


\(^1\) Supported in part by NSF grant DMS 8800663.
\(^2\) Another counterexample was found later, and independently, by John Beebee.