Let $W(n; a, b, c) = (a^n + b^n - c^n)^2$. The question seems to be whether, for n sufficiently large, there is a ratio P of polynomials with positive rational coefficients such that

$$W(n; a, b, c) = P(W(n; a-1, b, c), \ldots)$$

where, as far as I understand it, the variables are $W(n-m; a-i, b-j, c-k)$ where (m, i, j, k) run over a finite set of 4-tuples of integers with the restriction that:

(1) $m \geq 0$.
(2) If $m = 0$, then i, j, and k are not *all* zero. (Perhaps they are even required to be non-negative.)

Suppose that such a formula existed. Choose n to be some large odd integer, and let $a = b = 1$ and $c = 2^{1/n}$ considered as real numbers (so one can apply inequalities). The RHS is zero, and so it follows that $W(n-m; a-i, b-j, c-k) = 0$ for some (m, i, j, k). Equivalently

$$(2^{1/n} - k)^{n-m} = (1-i)^{n-m} + (1-j)^{n-m}.$$

Since the powers $2^{i/n}$ are linearly independent over $\mathbf{Q}$ for $i < n$, the LHS is irrational unless $k = 0$ and $m = 0$. Thus we have

$$2 = (1-i)^n + (1-j)^n$$

which (when n is odd) is easily seen only to occur for $i = j = 0$. But now we have shown $(m, i, j, k) = (0, 0, 0, 0)$ which is forbidden.