Binary Tree Jump Limit Theorem

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We have the biviarate generating function

$$H(q,x) = -\frac{-qx + x - 1 + \sqrt{q^2x^2 - 2q\,x^2 - 2qx + x^2 - 2x + 1}}{2qx}$$

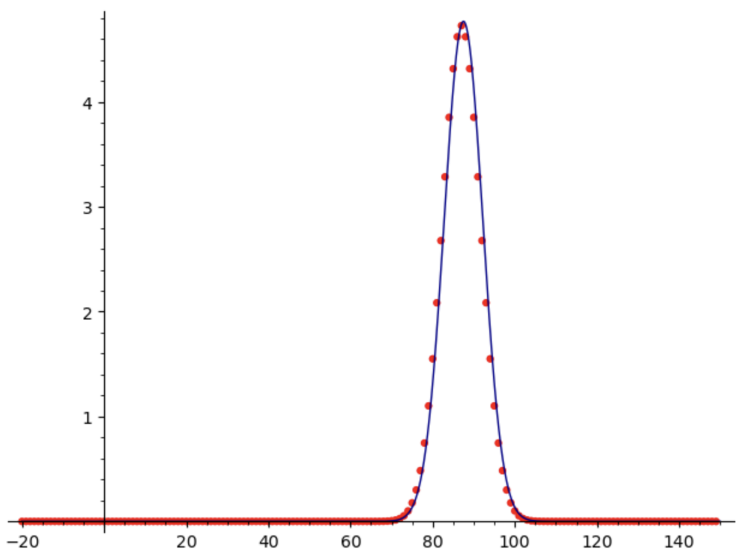
for the jump statistic. Writing

$$H(e^s,t) = A(s)\left(1 - \frac{t}{r(s)}\right)^{1/2} + B(s,t)\left(1 - \frac{t}{r(s)}\right)^{3/2}$$

for $A(s) = e^{-3s/4}\left(\mathrm{e}^{s/2} + 1\right)$ and $r(s) = (e^{s/2} + 1)^{-2}$, the results of Bender and Richmond [1] imply (after some analytic bounding on B) a central limit theorem and, with a bit more work, should give an LCLT

$$[q^sx^n]H(q,x) \approx \frac{4^n}{n^2} \cdot \frac{2}{\pi} \exp\left(-\frac{4}{n}\left(s - \frac{n}{2}\right)^2\right)$$

as $n \to \infty$. As a check, we plot of the series coefficients $c(s) = [q^sx^{175}]H(q,x)$ compared to the expected distribution. Proving such limit theorems is automated (and implemented in Sage) for many multivariate rational generating functions [2] but automating this for algebraic generating functions is ongoing work.



References

[1] Edward A. Bender and L. Bruce Richmond. "Central and local limit theorems applied to asymptotic enumeration. II. Multivariate generating functions". In: *J. Combin. Theory Ser. A* 34.3 (1983), pp. 255–265.

[2] Stephen Melczer and Tiadora Ruza. "Central Limit Theorems via Analytic Combinatorics in Several Variables". In: *Electron. J. Combin.* 31.2 (2024), Paper No. 2.27.