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Professor Doron Zeilberger
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Dear Doron,

Lemma 1 of your paper with Gillis in *Eur. J. Combinatorics* (1983) is a special case of a bijection I hadn't seen before. I'll try to paraphrase here the general idea:

Let L and H be disjoint sets of letters (low and high). Given a word $\alpha \in (L \cup H)^*$ containing l letters of L and h of H, map it into two words (α_L, α_H) as follows: α_L is α with all elements of H replaced by $\sqcup$, then all blanks in the last h positions are erased; α_H is α with all elements of L replaced by $\sqcup$, then all blanks in the first l positions are erased. It follows that α_L and α_H contain the same number of blanks, say k; also the last k letters of α_L and the first k of α_H are nonblank. Conversely, given any words $\alpha_L \in (L \cup \{\sqcup\})^*$ and $\alpha_H \in (H \cup \{\sqcup\})^*$ having these properties, we can uniquely reconstruct α by filling in the blanks.

Was this construction original with you and Gillis, or "well known" at the time? I want to assign proper credit.

Cordially,

Donald E. Knuth
Professor

DEK/pw

P.S. After writing the above, I seem to have found a generalization of Lemma 2 also. Please see the next page.

P.P.S. Do you know Gillis's full name? He seems to have disappeared from *Math Reviews* after 1984; there are many J. Gillises in the world and I hope to identify him more precisely in the index to my book.

P.P.P.S. Did I mention to you that the 'J. C. P. Miller recurrence' appears in both of Euler's calculus books (1748 and 1755), and is featured quite prominently in the latter?