Dr. Z.'s Oct. 2, 2017, RUMA Lecture quiz

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Everyone scoring perfectly will be listed in the webpage `http://www.math.rutgers.edu/~zeilberg/mamarim/mamarimhtml/anomalous.html`. The best entry will win a book prize.

1. In a certain country there are only two kinds of bills, a 9-dollar bill and a 77-dollar bill. Supposed that you want to pay for a cup of coffee that costs 1-dollar. What would you give the cashier, and what would he return to you as change? You **must** use the Extended Euclidean Algorithm.

Solve $9A+77B=1$

$77 = 9\cdot 8 + 5$

$9 = 5\cdot 1 + 4$

$5 = 4\cdot 1 + 1$

		A	B
r_1	77	1	0
r_2	9	0	1
r_3	5	1	−8 ($r_1 - 8r_2$)
r_4	4	−1	9 ($r_2 - r_3$)
r_5	1	2	−17 ($r_3 - r_4$)

Thus $1 = 2\cdot 77 - 17\cdot 9$

Give 2 $77 bills, get 17 $9 bills back

2. Find all single-digits x and y ($0 \le x \le 9$, $0 \le y \le 9$) such that the following **anomalous cancellation** is valid (in the usual base, base 10)

$$\frac{x\not{9}y}{5\not{9}2} = \frac{xy}{52}$$

You **must** use the algorithm presented in the lecture, but the part that requires the Extended Euclidean Algorithm may be done by inspection, if desired.

$$\frac{100x+30+y}{532} = \frac{10x+y}{52}$$

$$52(100x+30+y) = 532(10x+y)$$

$$5200x + 1560 + 52y = 5320x + 532y$$

$$1560 = 120x + 480y$$

$$13 = x + 4y$$

General solution!

$x = 13 - 4y$

$y = t$: (x, y):

$t=1$: $(9, 1)$

$t=2$: $(5, 2)$

$t=3$: $(1, 3)$

$\frac{931}{532} = \frac{91}{52} = 1.75$ ✓

$\frac{532}{532} = \frac{52}{52} = 1$ ✓

over

$$\frac{133}{532} = \frac{13}{52} = 0.25 \checkmark$$

Cont'
$= 77 - 9\times 8 - 9$
$+ 77 - 9\times 8$
$= \boxed{77\times 2 - 9\times 17}$

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77	17
x 2	x 9
154	153

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1. In a certain country there are only two kinds of bills, a 9-dollar bill and a 77-dollar bill. Supposed that you want to pay for a cup of coffee that costs 1-dollar. What would you give the cashier, and what would he return to you as change? You **must** use the Extended Euclidean Algorithm.

Sol: $77 = 9\times 8 + 5$

$9 = 5\times 1 + 4$

$5 = 4\times 1 + 1$

$\gcd(77,9) = \gcd(9,5)$
$= \gcd(5,4)$
$= \gcd(4,1)$

$1 = 5 - 4 = 5 - (9 - 5\times 1) = (77 - 9\times 8) - (9 - (77 - 9\times 8))$ Cont'

2. Find all single-digits x and y ($0 \le x \le 9$, $0 \le y \le 9$) such that the following **anomalous cancellation** is valid (in the usual base, base 10)

$$\frac{x\not{8}y}{5\not{8}2} = \frac{xy}{52}$$

You **must** use the algorithm presented in the lecture, but the part that requires the Extended Euclidean Algorithm may be done by inspection, if desired.

$2\overline{)532}\ \ 52$
$2\overline{)266}\ \ 26$
$133\ \ 13$

Sol: $\frac{100x+30+y}{532} = \frac{10x+y}{52}$.

$52(100x+30+y) = (10x+y)532$

$13(100x+30+y) = (10x+y)133$

$1300x + 390 + 13y = 1330x + 133y$.

$390 = 30x + 120y$.

$39 = 3x + 12y$

$13 = x + 4y$

$\frac{133}{-13}$
120

(impossible !!!)

$y=0,\ x=13$

$y=1\quad x=13-4=9$

$y=2\quad x=13-8=5$

$y=3\quad x=13-12=1$.

$\{(9,1), (5,2), (1,3)\}$ are solns.

$30 = 19 \times 1 + 11$

$19 = 11 + 8$

$11 = 8 \times 1 + 3$

$8 = 3 \times 2 + 2$

$3 = 2 \times 1 + 1$

$\gcd(30, 19)$
$= \gcd(19, 11)$
$= \gcd(11, 8)$
$= \gcd(8, 3)$
$= \gcd(3, 2)$.
$= \gcd(2, 1)$.

$1 = 3 - 2 \times 1$

$= (11 - 8 \times 1) - (8 - 3 \times 2)$

$= ((19 - 8) - 8 \times 1) - (8 - 3 \times 2)$

$=$

Dr. Z.'s Oct. 2, 2017, RUMA Lecture quiz

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Everyone scoring perfectly will be listed in the webpage `http://www.math.rutgers.edu/~zeilberg/mamarim/mamarimhtml/anomalous.html`. The best entry will win a book prize.

1. In a certain country there are only two kinds of bills, a 9-dollar bill and a 77-dollar bill. Supposed that you want to pay for a cup of coffee that costs 1-dollar. What would you give the cashier, and what would he return to you as change? You **must** use the Extended Euclidean Algorithm.

a	b	q	r
77	9	8	5
9	5	1	4
5	4	1	1

$5 = 77 - 8(9)$

$4 = 9 - 5 = 9 - (77 - 8(9)) = 9(9) - 77$

$1 = 5 - 4 = 77 - 8(9) - (9(9) - 77) = 77 - 8(9) - 9(9) + 77$

$= 2(77) - 17(9)$

You would give the cashier 2 77's, he would return 17 9's

2. Find all single-digits x and y ($0 \le x \le 9$, $0 \le y \le 9$) such that the following **anomalous cancellation** is valid (in the usual base, base 10)

$$\frac{x\,\beta y}{5\,\beta 2} = \frac{xy}{52}$$

You **must** use the algorithm presented in the lecture, but the part that requires the Extended Euclidean Algorithm may be done by inspection, if desired.

$$\frac{100x+30+y}{532} = \frac{10x+y}{52}$$

$$52(100x+30+y) = 532(10x+y)$$

$$5200x + 1560 + 52y = 5320x + 532y$$

$$1560 = 120x + 480y$$

$$13 = x + 4y$$

(x, y)
(9, 1)
(5, 2)
(1, 3)

by inspection

532 − 52 = 480

120 × 13 = 360 + 1200 = 1560

a	b	q	r
480	120	4	0

Dr. Z.'s Oct. 2, 2017, RUMA Lecture quiz

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1. In a certain country there are only two kinds of bills, a 9-dollar bill and a 77-dollar bill. Supposed that you want to pay for a cup of coffee that costs 1-dollar. What would you give the cashier, and what would he return to you as change? You **must** use the Extended Euclidean Algorithm.

$77 = 9\cdot 8 + 5$

$9 = 1\cdot 5 + 4$

$5 = 1\cdot 4 + 1$

$5 = 77 - 9\cdot 8$

$4 = 9 - 5 = 9 - (77 - 9\cdot 8) = 9\cdot 9 - 77$

$1 = 5 - 4 = 77 - 9\cdot 8 - (9\cdot 9 - 77)$

$1 = 2\cdot 77 - 9\cdot 17$

$154 - 153 = 1$

2. Find all single-digits x and y ($0 \le x \le 9$, $0 \le y \le 9$) such that the following **anomalous cancellation** is valid (in the usual base, base 10)

$$\frac{x\,\not{3}y}{5\,\not{3}2} = \frac{xy}{52}$$

$\frac{133}{532} = \frac{13}{52}$, $\frac{532}{\overline{532}} = \frac{52}{52}\,/\,\frac{931}{532} = \frac{91}{52}$

You **must** use the algorithm presented in the lecture, but the part that requires the Extended Euclidean Algorithm may be done by inspection, if desired.

$\frac{100x + 30 + y}{532} = \frac{10x + y}{52}$

$52(100x + 30 + y) = 532(10x + y)$

$5200x + 1560 + 52y = 5320x + 532y$

$1560 = 120x + 480y$

$156 = 12x + 48y$

$78 = 6x + 24y$

$39 = 3x + 12y \rightarrow 13 = x + 4y$

$x + 4y = 13$

$y = 3,\ x = 1$ initial

$x = 1 + 4t$

$y = 3 - t$

$t = 1$ $(x, y) = (5, 2)$

$t = 2$ $(x, y) = (9, 1)$

$t = 0$ $(x, y) = (1, 3)$

Dr. Z.'s Oct. 2, 2017, RUMA Lecture quiz

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1. In a certain country there are only two kinds of bills, a 9-dollar bill and a 77-dollar bill. Supposed that you want to pay for a cup of coffee that costs 1-dollar. What would you give the cashier, and what would he return to you as change? You **must** use the Extended Euclidean Algorithm.

$ax+by=1$

$77x + 9y = 1$

a	b	q	r
77	9	8	5
9	5	1	4
5	4	1	1

$[77] - [9]\cdot 8 = [5]$

$[9] - (77 - [9]\cdot 8) = 4$

$[77] - [9]\cdot 8 - ([9] - [77] - [9]\cdot 8)$

$= [77] - 8\cdot[9] - [9] + [77] - [9]\cdot 8$

$= 2\cdot[77] - 17[9]$

Pay with 2 \$77 bills, they return 17 \$9 bills

2. Find all single-digits x and y ($0 \le x \le 9$, $0 \le y \le 9$) such that the following **anomalous cancellation** is valid (in the usual base, base 10)

$$\frac{x\ \not{8}y}{5\ \not{8}2} = \frac{xy}{52}$$

You **must** use the algorithm presented in the lecture, but the part that requires the Extended Euclidean Algorithm may be done by inspection, if desired.

$$\frac{100x+30+y}{532} = \frac{10x+y}{52}$$

$$52(100x+30+y) = 532(10x+y)$$

$$5200x + 1560 + 52y = 5320x + 532y$$

$$1560 = 120x + 480y$$

$$13 = x + 4y$$

Solutions to $x+4y=13$

$x=9,\ y=1$

$x=5,\ y=2$

$x=1,\ y=3$

Dr. Z.'s Oct. 2, 2017, RUMA Lecture quiz

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Everyone scoring perfectly will be listed in the webpage `http://www.math.rutgers.edu/~zeilberg/mamarim/mamarimhtml/anomalous.html`. The best entry will win a book prize.

1. In a certain country there are only two kinds of bills, a 9-dollar bill and a 77-dollar bill. Supposed that you want to pay for a cup of coffee that costs 1-dollar. What would you give the cashier, and what would he return to you as change? You **must** use the Extended Euclidean Algorithm.

a	b	q	r	
77	9	8	5	$5 = (77) - 8(9)$
9	5	1	4	$4 = (9) - 5 = (9) - \big((77) - 8(9)\big) = 9(9) - (77)$
5	4	1	1	$1 = 5 - 4 = \big((77) - 8(9)\big) - \big(9(9) - (77)\big) = 2(77) - 17(9)$

give: $2 \times \$77$ ($154) change: $17 \times \$9$ ($153)

2. Find all single-digits x and y ($0 \le x \le 9$, $0 \le y \le 9$) such that the following **anomalous cancellation** is valid (in the usual base, base 10)

$$\frac{x\,\not{8}y}{5\,\not{8}2} = \frac{xy}{52}$$

You **must** use the algorithm presented in the lecture, but the part that requires the Extended Euclidean Algorithm may be done by inspection, if desired.

$$\frac{100x + y + 30}{532} = \frac{10x + y}{52}$$

$$5200x + 52y + 1560 = 5320x + 532y$$

$$120x + 480y = 1560$$

$$x + 4y = 13$$

$$x = 13 - 4t,\ y = t$$

x	y
9	1
5	2
1	3

$$\frac{931}{532} = \frac{91}{52} = \frac{7}{4}$$

$$\frac{532}{532} = \frac{52}{52} = 1$$

$$\frac{133}{532} = \frac{13}{52} = \frac{1}{4}$$

Dr. Z.'s Oct. 2, 2017, RUMA Lecture quiz

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Everyone scoring perfectly will be listed in the webpage `http://www.math.rutgers.edu/~zeilberg/mamarim/mamarimhtml/anomalous.html`. The best entry will win a book prize.

1. In a certain country there are only two kinds of bills, a 9-dollar bill and a 77-dollar bill. Supposed that you want to pay for a cup of coffee that costs 1-dollar. What would you give the cashier, and what would he return to you as change? You **must** use the Extended Euclidean Algorithm.

a	b	q	r
77	9	8	5
9	5	1	4
5	4	1	1
4	1	4	0

$5 = 1(77) - 9(8)$

$4 = 9 - 5(1) = 9 - [77(1) - 9(8)]$

$1 = 5 - 4$

$= 2(77) - 9(17)$

So, 2 bills of 77 $ and 17 bills of 9 $

2. Find all single-digits x and y ($0 \le x \le 9$, $0 \le y \le 9$) such that the following **anomalous cancellation** is valid (in the usual base, base 10)

$$\frac{x\ 3y}{5\ 32} = \frac{xy}{52}$$

You **must** use the algorithm presented in the lecture, but the part that requires the Extended Euclidean Algorithm may be done by inspection, if desired.

$$\frac{100x + 30 + y}{532} = \frac{10x + y}{52}$$

$5200x + 1560 + 52y = 5320x + 532y$

$\Rightarrow 120x + 48y = 1560$

$\Rightarrow 20x + 8y = 160$

$\Rightarrow 5x + 2y = 32$

$x = 6$ any $y = 1$ is our soln.

suppose $y = t$, $x = (32 - 2t)/5$

$y = 1, x = 6$
$y = 6, x = 4$