A Generalization of Odlyzko's Conjecture: The Coefficients of $(1-q)^j/((1-q^{2n})...(1-q^{2n+2j}))$ Alternate in Sign

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Let N, a, b, i, j, r, n denote arbitrary non-negative integers, and let

$$(x)_N := \prod_{i=0}^{N-1}(1-q^i x)\ \ , G(a,b) := \frac{(q)_{a+b}}{(q)_a (q)_b}\ ,\ A := \{f(q)\,;\,(-1)^i f^{(i)}(0) \geq 0, 0 \leq i < \infty\}.$$

A. Odlyzko conjectured that $(1-q)^j/(q)_j \epsilon A$. This was proved in [1], where the more general fact that $(1-q)^j G(2j,r) \epsilon A$ was proved. Here we show that this last result also implies the statement of the title, that generalizes Odlyzko's conjecture (for even j) in a different direction. To wit: set $t = q^{2n}, N = 2j$, in the q-binomial theorem $1/(t)_{N+1} = \sum_{r>=0} G(N,r)t^r$, and then multiply by $(1-q)^j$. QED.

Reference

1. D.Stanton and D. Zeilberger, *The Odlyzko conjecture and O'Hara's unimodality proof*, Proc. Amer. Math. Soc. **107**(1989), 39-42.

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