Mathematical Genitalysis: A Powerful New Combinatorial Theory that Obviates Mathematical Analysis

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Dedicated to my beloved gurus, Paul Halmos and Steven Krantz

Abstract: A Powerful new combinatorial theory, superseding, and sometimes trivializing, mathematical analysis, is introduced. It is illustrated by an exact determination of Bloch's constant, a generalization of Kaczynski's boundary functions from one to several complex variables, and a generalization of Petryshyn's theorem about the variational solvability of elliptic boundary value problems at resonance from the quasi-linear case to the fully non-linear case.

Introduction

With the possible exception of topology (formerly called *analysis situs*), mathematical analysis is the deepest and most technically challenging area of mathematics. It is also wrought with many logical and foundational problems. In this *research report*, I will outline a new *combinatorial* theory, that I have christened *genitalysis*, that promises to simplify, and in many cases *trivialize*, most, and potentially all, of the current open problems in analysis.

Like many novel ideas, the central idea of genitalysis is embarrassingly simple. It is 'symbolic discretization', which is roughly the 'symbolic' analog of 'numerical discretization', that has been used in applied and computational mathematics at least since Euler, but that has become a powerful and mature theory only in the last thirty years, spurred, of course, by the amazing *number crunching* capabilities of modern electronic computers.

The idea behind numerical discretization is to approximate a continuous problem by its discrete analog, on a *grid* of a small spacing h. More efficient approximations can be achieved by *multigrid* methods.

The 20^{th} century was the century of *numerical* computations, and Fortran, and later Pascal, C, C++, and Java ruled. The 21^{th} century, by contrast, is going to be the century of *symbolic* computations, and Maple, Mathematica, Macsyma, and their likes will conquer the mathematical world. The idea behind 'symbolic discretization' is to solve *symbolically*, and hence *rigorously*, the discrete analog, and at the very end let the mesh-size(s) h, tend to zero. The issue of convergence, in all the examples that we have encountered so far, is routine, because of the explicit nature of the symbolic solution. It would be very interesting to have an *a priori* guarantee of convergence of our method.

Obviously, by the law of conservation of difficulty, the 'trivialization' comes at a price. Our method requires extensive computer resources, as well as efficient software engineering. A very preliminary *beta version*, a Maple package called `DIBRA6`, will soon be available from the author's website.

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The exact forms of the symbolic expressions at specific cases are extremely complicated (to a human), but also irrelevant, since the passage to the limit $h \to 0$ is performed *silently* and completely *automatically.*

But enough of generalities. Let's go to business. My choice of examples may seem idiosyncratic, and it is certainly subjective: these are the examples that we liked the best! We were also motivated by the fame of the two complex analysts and the one non-linear mathematician with whom these examples are associated.

The Exact Value of Bloch's Constant

André Bloch(1893-1948) is one of the most eminent complex analysts who has ever lived. In 1924, he showed that a covering surface over the w-plane obtained from a mapping $w = F(z) = z + \ldots$ that is one-to-one, conformal, and holomorphic in $|z| < 1$ always contains a *univalent* (schlicht) disk whose radius B is a positive number independent of F. The supremum of such constants is called *Bloch's constant*, $\mathcal{B}$. It was a major open problem to determine its exact value. Previously only lower and upper bounds were known. Using the present method, we determined that

$$\mathcal{B} = \frac{3^{1/4}\pi^{1/4}\Gamma(1/3)^{1/2}\Gamma(11/12)^{1/4}}{2^{7/8}\Gamma(1/4)^{1/2}\Gamma(1/12)^{1/4}} \quad .$$

Generalization of Kaczynski's Theorem to C^n

Another celebrated complex-analyst is T.J. Kaczynski(1942-). Bagemihl and Piranian gave an example of a harmonic function having a boundary function defined on $|z| = 1$ that is not of Baire class 0 or 1, and they asked whether there exists a *bounded* harmonic function having a boundary function defined on $|z| = 1$ that is not of Baire class 0 or 1. Kaczynski[K] gave a brilliant affirmative answer by means of an intricate and ingenious construction. Using his ideas, and symbolic discretization, and to Kaczynski's (possible) dismay, 100 hours of CPU time on a Cray 3, we were able to extend his construction to several complex variables.

Petryshyn's Theory of Generalized Topological Degree Generalized from Semilinear to Nonlinear Equations

Equally brilliant, but perhaps not as widely known, is the non-linear Rutgers analysts W.V. Pertryshyn(1929-). In a series of brilliant papers, that culminated in the monograph [P], that appeared in the very prestigious Tracts series of Cambridge University Press, Petryshyn developed a theory of topological degree for semilinear equations. Using our new method, and the package DIBRA6, we were able to extend his results from *semilinear* equations to (fully) *nonlinear* equations.

Conclusion

We hope to be able, in the future, to present many more examples. Meanwhile, let us note that the present approach, being essentially *empirical*, lends credence to the extremely influential and very important *Calculus Reform Movement* and *NCTM standards*. Of course, the proofs are completely rigorous, but *a posteriori* trivial and uninteresting. The crucial part is *finding* the proofs, and this is done empirically. It is regrettable that some otherwise very wise people cling so desperately and fanatically to the sinking ship of *proof*.

References

[K] T.J. Kaczynski, *Boundary functions and sets of curvilinear convergence for continuous functions*, Trans. Amer. Math. Soc. **141**(1969), 107-125.

[P] W.V. Petryshyn, *"Generalized Topological Degree and Semilinear Equations"*, Cambridge Tract in Mathematics **117**, Cambridge University Press, 1995.