Math Bite: A Generalization of a Curiosity that Feynman Remembered All His Life

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If a boy named Morrie Jacobs told him that the cosine of 20 degrees multiplied by the cosine of 40 degrees multiplied by the cosine of 80 degrees equaled exactly one-eighth, he would remember that curiosity for the rest of his life, and he would remember that he was standing in Morrie's father's leather shop when he learned it. — James Gleick ['Genius', p. 47]

Morrie Jacobs's identity is the special case $k = 3$, $a = 20^o$, of the following identity that follows by induction on k using $\sin 2b = 2 \sin b \cos b$, with $b = 2^{k-1}a$.

$$2^k \prod_{j=0}^{k-1} \cos(2^j a) = \frac{\sin(2^k a)}{\sin a} \quad .$$