ENUMERATION SMOOTH LEGO TOWERS

ABSTRACT

1. Introduction

In the following, we use the definitions from [1]. Recall that "Lego" is a construction toy of interlocking plastic building blocks. For our purpose, we will consider them as rectangular tiles of dimensions $1 \times a$ ($a \in \mathbb{N}$). A polyomino can be realized in terms of a Lego tower as follows: Assume that we have an infinite supply of Lego pieces of dimensions $1 \times a$. Then every floor of the tower contains a finite (horizontal) sequence of pieces separated by gaps. A vertical sequence of floors constitutes a polyomino if the resulting configuration is connected. All the Lego towers depicted in Figure 1 are polyominoes.

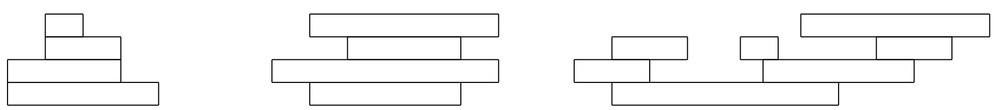

Figure 1. Examples of Lego towers.

Definition 1.1. *A k-*Lego tower *is a Lego tower with a finite number of floors, where each floor contains exactly one Lego piece of size $1 \times a$ with $1 \leq a \leq k$.*

For k-Lego towers, we introduce some more terminology. Given a k-Lego tower of two Lego pieces as described in Figure 2, we call $\ell \in \mathbb{Z}$ the *left overhang* and $\ell' \in \mathbb{Z}$ the *right overhang* of the second floor with respect to the first floor; note that both overhangs can be positive or negative (in Figure 2, $\ell < 0$ and $\ell' > 0$) .

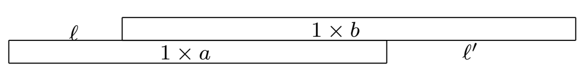

Figure 2. Lego tower with two Lego pieces with $b - \ell' - \ell = a$.

A Lego tower or k-lego tower is called *double side smooth* if the left overhang ℓ and the right overhang ℓ' satisfy $\ell, \ell' \in \{-1, 1, 0\}$, and is called *left side smooth* (respectively, *right side smooth*) if the left overhang ℓ (respectively, the right overhang ℓ') satisfy $\ell \in \{-1, 1, 0\}$ (respectively, $\ell' \in \{-1, 1, 0\}$).

2. Enumeration of left side smooth Lego tower adn k-Lego tower

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References

[1] D. Zeilberger, Automated counting of LEGO towers, *J. Differ. Equations Appl.* **5** (1999), 323–333.