MATH 151 (17-19), Dr. Z. , **Answers for the Practice For Midterm II**,

These posted answers refer to the Nov. 20, 7:30pm version. If you have an earlier version, please download the latest version of the Practice for Midterm II.

Disclaimer: Even I make mistakes. If you find any errors, please E-mail me: zeilberg at math dot rutgers dot edu . (Subject: MathIsFun). I will pay 1 dollar to the first person to detect any error (untlil Sunday Nov. 21, 8:00pm). Of course, if your answer diagrees, it is more likely that you made a mistake, so please check it three times before you report an error.

Nov. 21, 2004, 8:11am: Prashan found an error in 5(moderate-iii), it should be $2 + .03/e^2$. Here is the corrected version.

1. (warm-up) (a) $f(x) = 12x^2 + 6$, (b) $f(x) = \frac{20}{x^6}$, (c) $f(x) = \frac{-4}{x^2}$.

1. (moderate) (a) $f''(t) = -2\sin t - t\cos t$, (b) $f''(t) = e^t - 4t\cos t + (t^2 - 2)\sin t$, (c) $f''(s) = (s^2 + 4s + 2)e^s$, (d) $f''(x) = \frac{-4}{(x+3)^3} + 2e^x$.

1. (hard) (a)

$$f''(x) = \frac{6x(2 + x^6 + (1 - x^6)^{3/2})}{(\sqrt{1 - x^6})^3}$$

(b)

$$f''(x) = -\frac{1 + 2x\tan^{-1} x}{((1 + x^2)\tan^{-1} x)^2}$$

2. (warm-up) : Horiz. Asymp.: none. Vertical Asympt.: none. decreasing: $-\infty < x < 2$; increasing: $2 < x < \infty$; local max: none; local min: $(2, -3)$; concave up: $-\infty < x < \infty$; concave down: never; inflection point: none. (Sorry, I am too lazy to prepare a diagram on the computer or to scan).

2. (medium) Horiz. Asymp.: none. Vertical Asymp.: none. decreasing: $1 < x < 2$; increasing: $-\infty < x < 1$ AND $2 < x < \infty$; local max: $(1, 6)$; local min: $(2, 5)$; concave up: $3/2 < x < \infty$; concave down: $-\infty < x < 3/2$; inflection point: $(3/2, 11/2)$. (Sorry, I am too lazy to prepare a diagram on the computer or to scan).

2. (hard) Horiz. Asymp.: none. Vertical Asympt.: none. decreasing: $-\infty < x < 1$; increasing: $1 < x < \infty$; local max: none; local min: $(1, 4)$; concave up: $-\infty < x < 0$ AND $2/3 < x < \infty$; concave down: $0 < x < 2/3$; inflection points: $(0, 5)$ (horizontal inflection point!) and $(2/3, 119/27)$. (Sorry, I am too lazy to prepare a diagram on the computer or to scan).

3. (warm-up-i) Horiz. Asymp.: $y = 0$ (on both ends!). Vertical Asymp. $x = 1$. decreasing: $-\infty < x < 1$ AND $1 < x < \infty$; increasing: never. local max: none; local min: none; concave up: $1 < x < \infty$; concave down: $-\infty < x < 1$; inflection points: none. (Sorry, I am too lazy to prepare a diagram on the computer or to scan).

3. (warm-up-ii) Horiz. Asymp.: $y = 0$ (on both ends!). Vertical Asymp. $x = 2$. decreasing: $2 < x < \infty$; increasing: $-\infty < x < 2$; local max: none; local min: none; concave up: $-\infty < x < 2$ AND $2 < x < \infty$; concave down: never ; inflection points: none. (Sorry, I am too lazy to prepare a diagram on the computer or to scan).

3. (warm-up-iii) Horiz. Asymp.: $y = 0$ (on both ends!). Vertical Asymp. $x = -1$. decreasing: never; increasing: $-\infty < x < -1$ AND $-1 < x < \infty$; local max: none; local min: none; concave up: $-\infty < x < -1$; concave down: $-1 < x < \infty$; inflection points: none. (Sorry, I am too lazy to prepare a diagram on the computer or to scan).

3. (moderate-i) Horiz. Asymp.: $y = 1$ (on both ends!). Vertical Asymp. $x = 1$. decreasing: $-\infty < x < 1$ AND $1 < x < \infty$; increasing: never. local max: none; local min: none; concave up: $1 < x < \infty$ concave down: $-\infty < x < 1$ inflection points: none. (Sorry, I am too lazy to prepare a diagram on the computer or to scan).

3. (moderate-ii) Horiz. Asymp.: $y = 2$ (on both ends!). Vertical Asymp. $x = -1$. increading: $-\infty < x < -1$ AND $-1 < x < \infty$; decreasing: never. local max: none; local min: none; concave up: $-\infty < x < -1$ concave down: $-1 < x < \infty$ inflection points: none. (Sorry, I am too lazy to prepare a diagram on the computer or to scan).

3. (hard-i) Horiz. Asymp.: $y = 0$ (on both ends!); Vertical Asymp. $x = -2$ AND $x = 2$; increasing: never; decreasing: $-\infty < x < -2$ AND $-2 < x < 2$ AND $2 < x < \infty$; local max: none; local min: none; concave up: $-2 < x < 0$ AND $2 < x < \infty$ concave down: $-\infty < x < -2$ AND $0 < x < 2$. inflection points: $(0, 0)$. (Sorry, I am too lazy to prepare a diagram on the computer or to scan).

3. (hard-ii) Horiz. Asymp.: $y = 0$ (only from the right!); Vertical Asymp.: none; increasing:$-\infty < x < 1/2$; decreasing: $1/2 < x < \infty$; local max: $(1/2, 1/(2e))$; local min: none; concave up: $1 < x < \infty$ concave down: $-\infty < x < 1$ inflection points: $(1, 1/e^2)$. (Sorry, I am too lazy to prepare a diagram on the computer or to scan).

4. (war-up) R is decreasing at a rate of 5 Ohms per second.

4. (moderate-i) R is decreasing at a rate of 59/36 Ohms per second.

4. (moderate-ii) The radius is increasing at a rate of 1 centimeter per second.

4. (moderate-iii) The bottom of the ladder is moving away from the wall at a speed of 4/3 feet per second.

4. (hard) The water level is rising at a speed of $\frac{5}{24}$ meters per second.

5. (warm-up) 1.02.

5. (moderate-i) $(\sqrt{2}/2) \cdot (1.01)$

5. (moderate-ii) $3\frac{1}{108}$ or $\frac{325}{108}$.

5. (moderate-iii) $2 + .03/e^2$.

6. (warm-up) $x_2 = 2$.

6. (moderate-i) $x_2 = 4/3$, $x_3 = 20/63$.

6. (moderate-ii) $x_2 = 95/48$, $x_3 = 1152220351/532329100$.

7. (warm-up) It is continuous in $[0, 1]$ and differentiable in $(0, 1)$ (every polynomial is!, everywehere!). $c = 0.5$.

7. (moderate) It is continuous in $[0, 1]$ and differentiable in $(0, 1)$; $c = (3.2)^{1/4}$.

7. (hard) $\ln x$ is continuous and differentiable whenever $x > 0$, in particular in the interval $[e^2, e^3]$; $c = e^2(e - 1)$.

8. (warm-up) min value is 1; max value is 2.

8. (moderate) min value is 0; max value is 49.

9. (warm-up-i) 10 and 10.

9. (warm-up-ii) The maximal possible area is 1250 square feet, the dimensions are 25 by 50 feet.

9. (medium-i) $(500)^{1/3} \times (500)^{1/3} \times 2 \cdot (500)^{1/3}$.

9. (medium-ii) $(2, \sqrt{2})$.

9. (hard) radius is $(150)^{1/3}$ inches and height is $2 \cdot (150)^{1/3}$ inches.

9. (hard) $(550)^{1/3} \times (550)^{1/3} \times (1000)/(550)^{2/3}$ cm.

10. (warm-up) $x^2 + 3x + C$

10. (moderate-i)

$$f(x) = \frac{x^2}{2} - \frac{1}{x} + C$$

10. (moderate-ii)

$$f(x) = -3\cos x + 4\sin x + e^x + C$$

10. (hard-i)

$$f(x) = \frac{3x^2}{2} + 5\sin^{-1} x + C$$

10. (hard-ii)

$$f(x) = 3\frac{x^2}{2} + 5\tan^{-1} x + C$$

11. (warm-up) $f(x) = x^2$.

11. (moderate-i) $f(x) = 2x^2 + 2/x^3 + 2$.

11. (moderate-ii) $f(x) = -\cos x + 3$.

11. (hard-i) $f(x) = x^4/12 + x^2/2 + 2x/3 - 1/4$.

11. (hard-ii) $f(x) = x^4/24 - x^3/6 + 5x^2/4 - x/6 + 1/24$.