If A_n Has 6n Dyes in a Box, With Which He Has To Fling [at least] n Sixes, Then A_n Has An Easier Task Than A_{n+1}, at Eaven Luck

Doron ZEILBERGER

The probability of A_n succeeding, $1-\sum_{k=0}^{n-1}\binom{6n}{k}(\frac{1}{6})^k(\frac{5}{6})^{6n-k}$, from which the monotonicity is not obvious, can be rewritten (using `zeillim` in the package EKHAD accompanying [PWZ]) as:

$$1-\sum_{m=0}^{n-1}\frac{2\,(94500\,m^4+214830\,m^3+171573\,m^2+56243\,m+6250)\,(6m)!\,5^{5m+2}}{(5m+5)!m!\,6^{6m+5}} \quad ,$$

from which the monotonicity is obvious. This generalizes, from $n=1,2$, to general n, a statement first proved, in 1693, by Mr. Isaac Newton, in response to a question of Mr. Samuel Pepys.

References

[PN] S. Pepys and I. Newton, *correspondence*, reproduced in American Statistician, Oct. 1960, 27-30.

[PWZ] M. Petkovsek, H.S. Wilf, and D. Zeilberger, *"A=B"*, A.K.Peters, 1996. The accompanying Maple package EKHAD can be downloaded from the URLs given below.

Department of Mathematics, Temple University, Philadelphia, PA 19122, USA. `zeilberg@math.temple.edu`
`http://www.math.temple.edu/~zeilberg` `ftp://ftp.math.temple.edu/pub/zeilberg` .

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