Theorem 1: For all n > 0

Empirical Proof: True for $0 < n \le 10^{10000000000000000}$ (check!), hence true for all n.

Theorem 2: For all n > 0

Empirical Proof: True for $0 < n < 10^{10000000000000000}$ (check!), hence true for all n.

Definition: The Pisot sequence , E(x, y) where 0 < x < y are integers, is defined by the following nonlinear recurrence:

$$a_0 = x \quad , \quad a_1 = y \quad ,$$

and, for n > 1,

$$a_n := \left\lfloor \frac{a_{n-1}^2}{a_{n-2}} + \frac{1}{2} \right\rfloor \quad ,$$

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where, as usual, $\lfloor x \rfloor$ denotes the largest integer that is $\leq x$.