Research Announcement: The Transcendence of $e + \pi$ and $e\pi$

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There are $\aleph$ real numbers, while there are only $\aleph_0$ algebraic numbers. Hence the probability that a randomly chosen real number is algebraic equals $\frac{\aleph_0}{\aleph} = \frac{1}{\aleph}$. Nevertheless, it is notoriously difficult to prove that any *naturally occurring* number is irrational, let alone transcendental.

In 1978, Roger Apéry astounded the mathematical world with his proof of the irrationality of $\zeta(3)$. In his delightful account, Alf van der Poorten said([vdP], p. 197):

Indeed, proving the irrationality of $\zeta(2n+1)$, $n = 2, 3, \ldots$, *constitutes one of the outstanding problems of the theory (ranking with the arithmetical nature of* $\gamma := \lim_{n\to\infty}(1+\ldots+(1/n)-\log(n))$, *and of* $e+\pi$, $e\pi$, ..., *which are yet undetermined).*

The purpose of this note is to announce that Hermite's[H] celebrated result that e is transcendental, combined with an amazing (but apparently overlooked) statement of Goodwin[G], imply the transcendence of both $e + \pi$ and $e\pi$.

But even more interesting than the above implication is the way by which it was arrived, via *computer-generated deduction.*

We first developed a C-based meta-language, MISPAR, that has built-in number-theoretical deduction capabilities, that inputs suitably formatted statements about numbers (especially targeted to handle transcendence theory), and outputs new statements. Then, using ten diligent graduate students, many results that appeared in papers on the subject were entered in the appropriate format. Then we used a *genetic algorithm* to deduce million of new results, most of them either trivial or uninteresting (or both!).

Then we made a long list of open problems. Whenever the computer made a new deduction, it was compared against the statements in the list, looking for possible matches.

While we sure hoped to obtain new *interesting* results, even in our wildest dreams we did not anticipate such a spectacular deduction.

We are sure that *MISPAR* would make many more interesting deductions in the future. The package itself, and implementation details, will be eventually published at the author's website `http://www.math.temple.edu/~zeilberg`.

References

[G] E. J. Goodwin, *Amer. Math. Monthly*, **1** (1894), 246-247.

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[H] C. Hermite, *Comptes Rend. Acad. Sci. Paris*, **77**(1873), 18-24, 74-79, 285-293.

[vdP] A. van der Poorten, *Math. Intell.*, **1**(1979), 195-203.