The Maximal number of floors a Building can have where you can tell the highest floor from where you can throw a glass ball without breaking it, if you have b glass balls and are allowed t throws is $\binom{t}{b} + \binom{t}{b-1} + \ldots + \binom{t}{1}$

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Indeed, let $F(b,t)$ be the quantity described in the title and let $G(b,t) := \binom{t}{b} + \binom{t}{b-1} + \ldots + \binom{t}{1}$. After the first throw, if the ball broke, the maximal number of floors that you can handle below it is $F(b-1,t-1)$, and if it didn't break, the maximum floors that you can handle above it is $F(b,t-1)$, so we have the recurrence $F(b,t) = F(b-1,t-1) + F(b,t-1) + 1$ with the initial condition $F(1,t) = t$. By the Pascal-Chu-defining recurrence for the binomial coefficients, $G(b,t) = G(b-1,t-1) + G(b,t-1) + 1$. Since $G(1,t) = t$, the statement follows by induction on b and t. $\square$

The strategy is clear. Throw a ball from floor $G(b-1,t-1)+1$. If it breaks explore (recursively) the $G(b-1,t-1)$ floors below, (with the remaining $b-1$ balls and $t-1$ throws). Otherwise, explore (recursively) the $G(b,t-1)$ floors above.

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