I Am Sorry, Richard Ehrenborg and Margie Readdy, About Your Two Conjectures, But One is FAMOUS, While The Other Is FALSE

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R. Ehrenborg and M. Readdy[ER] raised two conjecture. The first one was that a certain triple-sum indexed by permutations of length $n$ equals $1!3!5!\cdots(2n - 1)!$. I don’t remember the details right now, but I immediately recognized it as equivalent to a special case of the celebrated Mehla Integral Identity (that was “open” between 1965 and 1980, until one time Freeman Dyson mentioned it to Atle Selberg, and the latter immediately deduced it from his (then obscure, but now famous) 1944 integral).

The second conjecture was more interesting. It claimed that there are exactly $k!$ Down-Up involutions of length $2k$. Richard checked it for $k = 1, \ldots, 5$. Alas, using my Maple package EHRENBOURG, it turns out (type nDUI(12);), that there are only 716 Down-Up involutions of length 12, which is 4 short of $6!$.

For the sake of completeness, here are the first 24 terms of the enumerating sequence for Down-Up involutions:

$[1, 1, 1, 2, 3, 6, 11, 24, 51, 120, 283, 716, 1833, 4948, 13561, 38788, 112745, 339676, 1039929, 3283876, 10532747, 34717276, 116158851, 398257012]$. 

REFERENCE


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