## You Don't Have To Be an Einstein to Figure Out that Sara Should (Asymptotically) Eat

 $\frac{n}{3}+\frac{4}{27}+O(1 / n)$ Dove Bars,But you do Need to be an Ekhad (or any of its silicon brethren) to Figure Out that She Should Eat (Asymptotically)

$$
\begin{gathered}
\frac{n}{3}+\frac{4}{27}+\frac{1}{81 n}+\frac{5}{729 n^{2}}+\frac{23}{6561 n^{3}}+\frac{1}{6561 n^{4}}-\frac{281}{59049 n^{5}}-\frac{5855}{531441 n^{6}}-\frac{18691}{1594323 n^{7}}+\frac{245947}{14348907 n^{8}}+\frac{15502093}{129140163 n^{9}}+\frac{106690105}{387420489 n^{10}}+O\left(\frac{1}{n^{11}}\right) \\
\text { Dove Bars } \\
\text { Shalosh B. EKHAD }{ }^{1}
\end{gathered}
$$

First download the Maple package: http://www.math.rutgers.edu/~zeilberg/tokhniot/AsyRec . Then, go into maple, and type:
read AsyRec:
$\operatorname{expand}\left(\right.$ Asy (SumTools[Hypergeometric][Zeilberger] $\left(\left(\mathrm{k}^{*} \operatorname{binomial}(\mathrm{n}+\mathrm{k}, \mathrm{k}) * \operatorname{binomial}\left(\mathrm{n}, \mathrm{n}-2^{*} \mathrm{k}\right)+\mathrm{k}^{*} \operatorname{binomial}(\mathrm{n}+\mathrm{k}-\right.\right.$ $1, \mathrm{k}-1) * \operatorname{binomial}(\mathrm{n}, \mathrm{n}-2 * \mathrm{k}+1)) / \operatorname{binomial}(2 * \mathrm{n}, \mathrm{n}), \mathrm{n}, \mathrm{k}, \mathrm{N})[1], \mathrm{n}, \mathrm{N}, 10) / 3)$;
and you would get the answer in the title, that improves from $O(1 / n)$ to $O\left(1 / n^{11}\right)$ the asymptotic formula, derived by clever (human) arguments by D.M. Einstein, C.C. Heckman, and T.S. Norfolk (Amer. Math. Monthly 116 (2009), 831-835). Of course, with a few more seconds of computations, you can get $O\left(1 / n^{30}\right)$ and beyond, but do you really care? Sara would get a terrible tummy ache way before even the $O\left(1 / n^{11}\right)$ asymptotics will start to be useful (and probably even before the original, humanly-derived, $O(1 / n)$ asymptotics).

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