Review of Dave Bressoud's "Proofs and Confirmations"

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In *Proofs and Confirmations: the Story of the Alternating Sign Matrix Conjecture*, Dave Bressoud has created a beautiful new genre of mathematical exposition. It is neither popular mathematics, nor textbook, nor research monograph, nor problem book. It is all these and much more: a historical novel, a detective story, and, implicitly, a philosophical manifesto. Yet the mathematics is deep, and all the proofs are complete.

Dave Bressoud belongs to that tiny set of mathematicians—Polya and Knuth come to mind—who are first-rate researchers *and* masters of expository writing. Bressoud is also very passionate about teaching and very active in mathematics education. He is one of the few moderate and sober voices in the fierce battle between extreme reformists (like Gleason and Hughes Hallet) and hard-nosed counter-reformists (like Andrews and Askey, to whom the book is dedicated). He believes that we need change, but that we should not *lose the very essence of mathematics, which is inextricably tied to proofs.*

Unfortunately, proofs have recently acquired a bad name. They smack of authoritarian despotism, and their quaint claim for absolute certainty seems anachronistic in this postmodern age of relativism and deconstruction. Of course, this is partly our fault. We mathematicians just keep teaching the same old way and losing the best and the brightest, first to physics and now to computer science. According to some people such as Gregory Chaitin, proofs are indeed *passé*, and we should start doing quasi-empirical mathematics; that is, we humans can only prove *trivial* theorems, and if we want to get a glimpse of non-trivial mathematical worlds, we have to make do with quasi-certainty.

Most scientists do not worry or care about (mathematically rigorous) proofs, and they often ridicule mathematicians as eccentric and pedantic fanatics. Flourishing areas of physics like quantum field theory and the renormalization group method are non-rigorous, and physicists view with indifference and mild amusement efforts by mathematicians to make them rigorous. In recent years, physics has helped mathematics much more than vice-versa by infusing new ideas and techniques, for example, the Seiberg-Witten equations that have revolutionized lower-dimensional topology. Another great discovery by physicists that was very influential in mathematics is the Yang-Baxter equation. One of its consequences, the Izergin-Korepin formula, turned out to be crucial for the proof of the Alternating Sign Matrix Conjecture.

But, for better or for worse, proofs are *the very essence of mathematics*, and if we want to keep the mathematical flame alive and attract to our profession the most mathematically talented, rather than the computer-science drop-out, we have to find ways to present proofs, and the *activity of proving*, in a way that will excite students and make them want to become mathematicians.

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That's why we desperately need books like this one. It conveys not only the mathematics but the excitement of doing the two major mathematical activities: conjecturing and proving. Bressoud chose the Alternating Sign Matrix Conjecture and its proof as a medium to *prove* his message: that mathematics *and proofs* are lively and exciting activities. The story itself and the mathematics along the way are charmingly told. You will encounter many interesting main characters with their even more interesting mathematics: Euler, Jacobi, Sylvester, MacMahon, Schur, Andrews, Mills-Robbins-Rumsey, Stanley, Baxter, Izergin-Korepin, Kuperberg, and, of course, Zeilberger. You will also get to know the work of supporting actors like Dodgson, Gessel, Viennot, Askey, and Krathenthaler. Also, very modestly and understatedly, Bressoud's own technical contributions make cameo appearances.

I almost forgot—what are alternating sign matrices, and what is the conjecture about them? ASM's were discovered by Howard Rumsey and Dave Robbins in their elegant extension of Dodgson's method for evaluating determinants. In an ASM, every entry is either 0, -1 or 1, and in every row and column the non-zero entries alternate, starting and ending at 1, i.e. a line can either have just one 1 and all the rest 0, or have the pattern $1, -1, 1$, interspersed with zeros, or the pattern $1, -1, 1, -1, 1$ sprinkled with zeros, etc. Here is an example of an ASM:

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

If there are no -1's, then we have permutation matrices, of which there are $n!$. In 1980, William Mills, Dave Robbins, and Howard Rumsey conjectured that the number of $n \times n$ ASMs is

$$\frac{1!4!7! \cdots (3n-2)!}{n!(n+1)! \cdots (2n-1)!}.$$

All of this and much more, including the ultimate proof, is grippingly described in this book.

The book is also visually very attractive. In particular, Greg Kuperberg's beautiful rendition of an ASM as a zigsaw puzzle adorns the cover, there are many clear diagrams, and the photographs of the main players in the drama add a warm human touch.

This book is destined to be a classic. It should be on the shelf of every mathematician, regardless of specialty, since the larger message that it conveys is universal. It is also ideal as a textbook for an innovative course for math majors or graduate students, and is already used very successfully as such, for example, at the University of Wisconsin. Make sure to buy a copy while the first edition lasts, and have Dave sign it next time you see him at a National meeting (he usually shows up). One day you, or your heirs, will thank me for this advice.