A Conjectured Explicit Determinant Evaluation whose Proof Would Make Us Happy (and the OEIS Richer)

By

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Abstract: We conjecture a certain explicit determinant evaluation, whose proof would imply the solution of a certain enumeration problem that we have been working on, and that we find interesting. We are pledging \$500 to the OEIS Foundation (in honor of the prover!) for a proof, and \$50 (in honor of the disprover or his or her computer) for a disproof, as well as (in the affirmative case only) a co-authorship in a good enumeration paper, that would immediately bequest a Zeilberger-number 1, an Erdös number $\leq$ 3, an Einstein number $\leq$ 4, and numerous other prestigious numbers.□

In order to complete the proof of a certain enumeration problem that we have been working on for the last few weeks, we need a proof of the following conjecture.

Let d be a positive integer, and let $M = M(d)$ be the following $2d \times 2d$ matrix with entries in $\{-1, 0, 1\}$. For $1 \leq a \leq 2d$ and $1 \leq b \leq d$,

$$M_{a,2b-1} = \begin{cases} 1 & \text{if } a = 2b \text{ ;} \\ -1 & \text{if } a = 3b+1; \\ 0. & \text{otherwise} \end{cases}$$

$$M_{a,2b} = \begin{cases} 1 & \text{if } a = 2b-1 \text{ ;} \\ -1 & \text{if } a = b-1; \\ 0. & \text{otherwise} \end{cases}$$

Conjecture: For every positive integer d, the following is true:

$$\det M(d) = (-1)^d \quad .$$

Comments:

1. This conjecture came up in our current work in enumerative combinatorics. Shalosh B. Ekhad kindly verified it for $d \leq 200$. We have no idea how hard it is, and it is possibly not that hard, but right now we are busy with other problems. We believe that the powerful and versatile techniques of Krattenthaler[K1][K2] may be applicable, and possibly the computer-assisted approach described in [Z] and already nicely exploited in [KKZ] and [KT].

2. The Short Maple code in: `http://www.math.rutgers.edu/~zeilberg/tokhniot/DetConj` defines the matrix $M(d)$ (procedure `M(d)`) and procedure `C(N)` verifies it empirically for all $d \leq N$. So far `C(200);` returned `true`.

3. We are offering to donate \$500 to the OEIS Foundation for a proof and \$50 for a disproof, with an explicit statement that the donation is in honor of the prover (or disprover).

References

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[KKZ] Christoph Koutschan, Manuel Kauers, and Doron Zeilberger , *A Proof Of George Andrews' and David Robbins' q-TSPP Conjecture*, Proceedings of the National Academy of Science, **108#6** (Feb. 8, 2011), 2196-2199.

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