Strategies for the cannibals and missionaries puzzle

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We consider the problem of determining whether there exists a solution given values for the following parameters:

- M, the number of missionaries, should be at least 1
- C, the number of cannibals, should be at least 1
- B, the boat size, should be at least 2
- d, if both cannibals and missionaries are present (on either side of the river or in the boat) then the number of missionaries must be at least the number of cannibals plus d.

1 Strategies

Here are some strategies for solving the puzzle.

Two Boat: This strategy works for any boat size, and never requires missionaries and cannibals to be in the boat at the same time. A single missionary can be sent across by sending 2 across and then having one come back with the boat. In this manner, we send as many missionaries across as possible, 1 at a time, and then we alternate sending a cannibal across with sending a missionary across. Without violating the rule, we can send over $M-C-d-1$ missionaries across at the start, and bring the boat back. Then we can no longer send over 2 missionaries, so we send over two cannibals with the intent of having 1 come back with the boat. This will not violate the rule as long

$$(M-C-d-1) \geq 2+d$$

$$M-C \geq 2d+3$$

From there it is easy to alternate sending over missionaries with cannibals until the puzzle is complete.

Big Boat: If the boat is big enough relative to the number of Cannibals, we can send all of the missionaries across before sending any cannibals across. Similar to Two Boat, we start by sending $M-C-d-1$ missionaries across and bringing the boat back. The number of missionaries remaining is then $C+d+1$. If

$$B \geq C+d+1$$

, we can proceed to send the rest of the missionaries over, send $B-1$ back, use them to ferry a single cannibal across, send the cannibal back, and then the cannibals can all ferry themselves across.

Big Boat 2: If the boat is so big that it can carry all the missionaries in it, that also works. Send 2 cannibals across, send 1 back, send all the missionaries over, send 1 cannibal back, send the rest of the cannibals across.

$$B \geq M, C \geq 2$$

Split Cannibals: In this strategy we send half of the cannibals over, and then all the missionaries over. It is similar to Big Boat in that it only works if the Boat is large compared to half the number of cannibals. The strategy is slightly different depending on if there is an even number or odd number of cannibals. First consider the even case. We send over half the cannibals. Then we can send a boat of $C/2+d+1$ missionaries. This requires

$$B > C/2+d+1$$

and

$$M-(C/2+d+1) \geq C/2+d$$

which simplifies to

$$M-C \geq 2d+1$$

Then the missionaries can ferry themselves across, and then the cannibals can ferry themselves across.

In the odd case, we send $\lceil C/2 \rceil$ cannibals across, then $\lceil C/2 \rceil+d+1$ missionaries across, and we can proceed similarly if

$$B > \lceil C/2 \rceil + d + 1$$

and

$$M-(\lceil C/2 \rceil + d + 1) \geq \lfloor C/2 \rfloor + d$$

which simplifies to

$$M-C \geq 2d+1$$

. However, it is still sometimes possible if

$$B = \lfloor C/2 \rfloor + d + 1$$

This is a current area of research.

Simultaneous Ferry: Here we start by sending d missionaries over, and then we repeated send $d+1$ missionaries and 1 cannibal over, and d missionaries back. Since cannibals and missionaries are in all 3 places at once, this requires

$$M - C \geq 3d$$

in addition to

$$B \geq d + 2$$

2 Sufficient conditions for $d \geq 1$

Strategy	Condition 1	Condition 2
Two Boat	$M - C \geq 2d + 3$	
Big Boat 1	$B \geq C + d + 1$	
Big Boat 2	$B \geq M$	$C \geq 2$
Split Can	$M - C \geq 2d + 1$	$B > \lceil C/2 \rceil + d + 1$
Simul	$M - C \geq 3d$	$B \geq d + 2$

Of course the condition that $M - C \geq d$ is assumed.

3 $d = 0$ case

In this case it does not make sense to use the previous approaches. We first give a simple strategy if $d = 0$ and $M > C$.

$d = 0, M > C$: Send over a boat with 1 missionary and 1 cannibal. Send 1 cannibal back. Send over 1 missionary and 1 cannibal. Send 1 missionary back. Repeat this strategy until all the cannibals are across, then have the missionaries ferry the remaining missionaries across.

When $M = C$ and $B \geq 4$, another simple strategy applies.

$d = 0, M = C, B \geq 4$: Send over 2 missionaries and 2 cannibals. Send 1 missionary and 1 cannibal back. Repeat until everyone is across.

The remaining cases are when $M = C$, and either $B = 2$ or $B = 3$. For $B = 2$ it turns out to be doable when $M \leq 3$ and for $B = 3$ it turns out to be doable when $M \leq 5$.

4 Sufficient conditions?

Are the above conditions necessary for the riddle to be solved? Not entirely. One example is that sometimes the Split cannibals strategy can still be applied when the condition is not satisfied. With more computation power, we should get a better idea of any potential cases where a strategy not described can be used.