

The Sagan-Savage Lucas-Catalan Polynomials Have Positive Coefficients

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In the last few minutes of Bruce Sagan's wonderful talk[1] about his joint work with Carla Savage[2] (whose notation I shall use), he mentioned that they can prove that $\frac{1}{\{n+1\}} \{2n\}$ are polynomials with integer coefficients, and *conjectured* that they are in fact polynomials with *positive* coefficients.

But this follows immediately from the identity $\frac{1}{\{n+1\}} \{2n\} = \{2n-1\}_{n-1} + t \{2n-1\}_{n-2}$ that after routine cancellations is equivalent to $\{2n\} = \{n+1\}\{n\} + t\{n-1\}\{n\}$, that is the case $m = n$ of Lemma 2.1 of [2]. \square

References

[1] Bruce Sagan, *Combinatorial Interpretations of Binomial Coefficient Analogues Related to Lucas Sequences*, talk at the Rutgers University Experimental Mathematics seminar on Dec. 9, 2010, videotaped by Edinah Gngang. <http://www.youtube.com/watch?v=Fdn890jg2U0> .

[2] Bruce Sagan and Carla Savage, *Combinatorial Interpretations of Binomial Coefficient Analogues Related to Lucas Sequences*, *Integers* **10** (2010), 697-703, A52. <http://arxiv.org/abs/0911.3159> .

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