Aufgabe VII.47 of Pólya-Szegö Immediately Implies Dave Robbins's Multi-Integral Evaluation

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Exercise VII.47 of [PS] (brought to my attention by Richard Stanley),

$$\sum_{\pi \in \mathcal{S}_k} \mathrm{sgn}(\pi) \cdot \pi \left[\frac{x_1 x_2^2 \dots x_k^k}{(1-x_k)(1-x_k x_{k-1}) \cdots (1 - x_k x_{k-1} \dots x_1)} \right] = \frac{x_1 \cdots x_k \prod_{1 \le i < j \le k} (x_j - x_i)}{\prod_{i=1}^k (1-x_i) \prod_{1 \le i < j \le k} (1 - x_i x_j)} \quad ,$$

(that is easily proved by induction on k and Lagrange Interpolation), immediately implies the main result of [R], upon setting $x_i := q^{a_i}$, multiplying by $(1-q)^k$, and letting $q \to 1$.

References

[PS] George Pólya and Gabor Szegö, *Aufgaben und Lehrsätze aus der Analysis*, Julius Springer, Berlin, 1925.

[R] David Robbins, *An application of Okada's Minor Summation Formula to the Evaluation of a Multiple Integral*, xxx archives (`http://xxx.lanl.gov`), `math.CO/9805108`, 23 May 1998, (3 pages).

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`http://www.math.temple.edu/~zeilberg` . May 28, 1998. Exclusive to the author's website and the xxx archives.